Measurement of the spin structure functions $g_{1}^{p,d,n}$ and the gluon helicity $\Delta g/g$ at HERMES

Achim Hillenbrand
(DESY)

for the HERMES collaboration

• $g_{1}$ from inclusive longitudinal double-spin asymmetries
• $\Delta g/g$ from high-$p_{t}$ hadrons
**g₁: Inclusive DIS**

**HERA:**
\[ e^+ @ 27.6 \text{ GeV} \]
\[ P_B \sim 53\% \]

**long. pol. undiluted gas target:**
- H (P\(_z\) \sim 76\%, 85\%)
- D (P\(_z\) \sim 84\%)

**Cross section \to structure functions**

\[ g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \left[ \Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) \right] \]

in LO QCD

\[ \Delta q = q_{\uparrow \uparrow} - q_{\downarrow \downarrow} \]
**Measured Inclusive Asymmetries**

\[
P_{zz} = 0.83 \pm 0.03 \quad A_{zz} \sim 0.01
\]

\[
\sigma = \sigma_{\text{unpol}} \left[ 1 + P_B P_z A_{\parallel} + \frac{1}{2} P_{zz} A_{zz} \right]
\]

Deuterium

\[
A_{\parallel} = \frac{\sigma \leftarrow \rightarrow - \sigma \leftarrow \leftarrow}{\sigma \leftarrow \leftarrow + \sigma \leftarrow \rightarrow} = \frac{1}{P_B P_z} \cdot \frac{N \leftarrow \leftarrow - N \leftarrow \rightarrow}{N \leftarrow \leftarrow + N \leftarrow \rightarrow}
\]

inclusive asymmetry:

\[
g_1(x, Q^2) = \frac{1}{1 - \frac{y}{2} - \frac{1}{4} y^2 \gamma} \left[ \frac{Q^4}{8\pi\alpha^2 y} \frac{\partial^2 \sigma_{\text{unpol}}}{\partial x \partial Q^2} A_{\parallel}(x, Q^2) + \frac{y}{2} \gamma^2 g_2(x, Q^2) \right]
\]

kinematic factors

param. meas. fac. param.
Unfolding of radiative effects

- Measured events have to be corrected for:
  - Background tail (radiation from (quasi)-elastic)
  - Radiation from DIS and detector smearing
- Event migration is simulated by Monte Carlo which includes a full detector description and a model for the cross section
- The approach is independent on the model for the asymmetry in the measured region

\[ \frac{\delta A_{\text{unfolded}}}{\delta A_{\text{raw}}} \]

detector smearing

\[ x_{bj} \]

Corrected introduces statistical correlation
• HERMES points: stat. and syst. errors added in quadrature
  ▶ Stat. uncertainties are diagonal elements of covariance matrix
  ▶ Syst. uncertainties dominated by target and beam polarisation

• Deuteron data:
  ▶ Most precise published data in valence x region

• Proton data:
  ▶ Stat. precision comparable to previous data

• $Q^2$ different from SMC/COMPASS
$g_1$: Results for $p$ and $d$
$g_1^n$ : Neutron results

$g_1^n = \frac{2}{1 - \frac{3}{2} \omega_D} \cdot g_1^d - g_1^p$

$\omega_D = 0.05 \pm 0.01$

- $g_1^n$ negative everywhere except at very high $x$
- Low-$Q^2$ data tends to zero at low $x$
  - Contrary to SMC data at higher $Q^2$
\( g_1 : \text{Integrals} \)

\[
\Gamma_d^1 = \int dx \, g_1^d
\]

Assuming saturation in the deuteron integral:

\[ \rightarrow \text{Use only deuteron data!} \]

\[
\Gamma_1^d = \left(1 - \frac{3}{2} \omega_D\right) \frac{1}{36} \left[4a_0\Delta C_{\overline{MS}}^S + a_8\Delta C_{\overline{MS}}^{NS}\right]
\]

\[ a_0 \overline{MS} \equiv \Delta \Sigma \]

<table>
<thead>
<tr>
<th>in central uncertainties</th>
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<th>theor.</th>
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\( Q^2 = 5 \text{GeV}^2 \), NNLO in \( \overline{\text{MS}} \) scheme
\( g_1 : \text{Integrals} \)

\[
\Gamma^d_1 = \int dx \ g^d_1
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\( a_0 \) \( MS \) \( \Delta \Sigma \)

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Assuming saturation in the deuteron integral:

→ Use only deuteron data!

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Assuming saturation in the deuteron integral:

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\]

from hyperon beta decay
(a\(_8\)=0.586±0.031)

\[
\begin{align*}
\Delta u + \Delta \bar{u} &= \frac{1}{6} \left[2a_0 + a_8 + 3a_3\right] \\
\Delta d + \Delta \bar{d} &= \frac{1}{6} \left[2a_0 + a_8 - 3a_3\right] \\
\Delta s + \Delta \bar{s} &= \frac{1}{3} \left[ a_0 - a_8 \right]
\end{align*}
\]

from neutron beta decay
(a\(_3\)=1.269±0.003)

\( Q^2 = 5\text{GeV}^2 \), NNLO in \( \overline{\text{MS}} \) scheme
How to measure $\Delta g$?

- Indirect from scaling violation of $g_1$
  - for fixed target experiment small $x$-$Q^2$ lever arm

- Direct measurement:
  - via processed with the gluon of the nucleon in the initial state
  - Photon-Gluon-Fusion (PGF)

- Measure longitudinal double-spin asymmetries of high-$p_t$ hadrons vs $p_t$ for:
  - “anti-tagged” data: scattered lepton not in acceptance ($p_t$ with respect to the beam axis)
  - “tagged” data: scattered lepton detected
  - inclusive pairs of charged hadrons

\[
\begin{align*}
\gamma^* & \rightarrow q \\
g & \rightarrow \bar{q}
\end{align*}
\]
Extraction Method

• Measured asymmetry is an incoherent superposition of different sub-process asymmetries:

\[ A_{||}^{\text{meas}}(p_t) = \sum_i f_i A_{||}^i = f_{\text{Sig}} A_{||}^{\text{Sig}} + f_{\text{Bg}} A_{||}^{\text{Bg}} \]

\[ f_i = \frac{\sigma_i}{\sigma_{\text{tot}}} \]

Signal: Gluon of the nucleon in the initial state

\[ A_{||}^{\text{Sig}}(p_t) \propto \int_{x(p_t)} dx \sigma(x, p_t) \hat{a}(x, p_t) \frac{\Delta g(x)}{g(x)} \]

Background: all other sub-processes → MC

• Two methods to extract \( \Delta g/g \)
  
  ▶ Method I: \( \Delta g(x)/g(x) = \text{const} \)
  
  ▶ Method II: \( \Delta g(x)/g(x) = x(1 + p_1(1-x)^2 + p_2(1-x)^3) \)
Models and Assumptions

• MC model
  ‣ PYTHIA 6.2, tuned and adapted for HERMES data
    fragmentation process, exclusive $\rho^0$ cross section (VMD)

• provides
  ‣ relative contributions $f_i$ of the background and signal subprocesses
    in the relevant $p_t$ range
  ‣ background asymmetries and the hard subprocess asymmetries of
    the signal processes
    - weight calculated for every MC event
  ‣ PDFs (unpol/pol): CTEQ5L/GRSV2000 (nucleon), SaS2/GRS (photon)
    - Asymmetry assumptions for soft processes (soft VMD)

• Vary PDFs/assumptions for syst. error
Fractions and Asymmetries

(anti-tagged data)

Sub-process fraction

Sub-process asymmetries
(using GRSV std.)

• DIS dominating at high $p_T$
• Signal processes are PGF and QCD2→2(g) (resolved photon)
• Background processes have (mostly) small and positive asymmetries
• |PGF| increasing with $p_T$, negative (for positive dg/g from GRSV)

Achim Hillenbrand
Asymmetries

\[ \Delta g/g(x) = 0 \text{ asymmetry is due to quarks only!} \]

Gluons become important for the cross section (asymmetry) above \( p_T \approx 1 \text{ GeV} \)

- **Anti-tagged data:**
  - Scattered lepton not in acceptance
  - \( p_T \) measured with respect to beam axis

- **Curves from MC**
  - Asymmetry model using:
    - \( \Delta g/g(x) = 0 \) : central
    - \( \Delta g/g(x) = -1 \) : upper
    - \( \Delta g/g(x) = +1 \) : lower
Δg/g: Method I

assume Δg(x)/g(x) const. over x: 

\[
\langle \frac{\Delta g}{g} \rangle = \frac{1}{f_{\text{Sig}} \langle \hat{a} \rangle} \left[ A_{\text{meas}} - f_{Bg} A_{\text{Bkg}} \right]
\]

- h⁺, h⁻ anti-tagged
  4 points between 1.05 < p_t < 2.5 GeV

- h⁺, h⁻ tagged
  1 point for p_t > 1 GeV

h pairs
  1 point for \( \sum p_t^2 > 2 \text{GeV} \)

- Results for different data samples agree within statistics
- Dominating sample: Deuteron antitagged
  ➞ Used for Method II

Proton

Deuteron

HERMES Preliminary

p_T (GeV)
**Δg/g: World data**

- **Black and blue curves:** pQCD fits to \( g_1 \)
- **Red curves (Method II):** fit \( Δg(x)/g(x) \) such that

\[
A_{\text{MC}}^/ \neq A_{\text{meas}}^/ 
\]

- **Sys. model uncert. dominating:**
  - PDFs
  - PYHTIA model

- **HERMES preliminary**
Conclusions

• HERMES has measured $g_1$ for proton and deuteron for $0.004 \leq x \leq 0.9$ and $0.18 < Q^2 < 20 \text{ GeV}^2$
• Proton data precision is comparable with CERN and SLAC
• Deuteron data is the most precise so far
• The deuteron integral is observed to saturate

$$a_0 = 0.330 \pm 0.011(\text{theor}) \pm 0.025(\text{exp.}) \pm 0.028(\text{evol})$$

• $\Delta g/g$ has been extracted using two different methods
• $\Delta g/g$ is likely small
• Method I: $\frac{\Delta g}{g}(x, \mu^2) = 0.078 \pm 0.034(\text{stat}) \pm 0.011(\text{sys} - \text{exp}) \pm 0.125(\text{sys} - \text{Models})$
• Method II: $\frac{\Delta g}{g}(x, \mu^2) = 0.071 \pm 0.034(\text{stat}) \pm 0.010(\text{sys} - \text{exp}) \pm 0.127(\text{sys} - \text{Models})$
Additional slides
Internal gas target: pol.: He, H, D  
unpol.: H₂, D₂, He, N₂, Ne, Kr, Xe  
Particle ID: TRD, Preshower, Calorimeter, Cerenkov (until 1997), RICH (since 1998)  
Reconstruction: $\Delta p/p < 2\%$, $\Delta \Theta < 1$ mrad
**g_1: Data Set and Binning**

<table>
<thead>
<tr>
<th>Target</th>
<th>Year</th>
<th>Luminosity (pb^{-1})</th>
<th>#</th>
<th>P_{Target} (%)</th>
<th>P_{Beam} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1996</td>
<td>12.6</td>
<td>670,000</td>
<td>75.9±3.2</td>
<td>~53±1.8</td>
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<tr>
<td>H</td>
<td>1997</td>
<td>37.3</td>
<td>2,800,000</td>
<td>85.1±3.2</td>
<td>~53±1.8</td>
</tr>
<tr>
<td>D</td>
<td>2000</td>
<td>138.7</td>
<td>10,900,000</td>
<td>85.1±3.2(+)</td>
<td>84.0±3.1(-)</td>
</tr>
</tbody>
</table>

\[ Q^2 = 0.18 \text{ GeV}^2 \]
\[ Q^2 = 1 \text{ GeV}^2 \]
\[ y = 0.91 \]
\[ \theta = 0.22 \text{ rad} \]
\[ \theta = 0.04 \text{ rad} \]
\[ W^2 = 3.24 \text{ GeV}^2 \]
$g_1^{p,d}$

**HERMES**

- $g_1^p$
- $g_1^d$

**COMPASS**

- $Q^2 < 1 \text{ GeV}^2$
- $Q^2 > 1 \text{ GeV}^2$

**EPS HEP 2007, Manchester, UK**
Fit results (Method II)

- Functions are polynomials with 1 or 2 free parameters
- Fix $\Delta g/g \to x$ for $x \to 0$ and $\Delta g/g \to 1$ for $x \to 1$ (Brodsky et al.)
- $|\Delta g(x)/g(x)| < 1$ for all $x$
- Difference between functions is systematic uncertainty

- Light shaded area: range of data
- Dark shaded area: center of gravity for fit