HERMES results on TMD measurements in SIDIS off a transversely polarized hydrogen target

Ami Rostomyan

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### quark structure of the nucleon

#### quark polarisation

<table>
<thead>
<tr>
<th>U</th>
<th>L</th>
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<td><strong>U</strong></td>
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<tr>
<td>$f_1$</td>
<td>$g_1$</td>
<td>$h_L^1$</td>
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<tr>
<td>number</td>
<td>helicity</td>
<td>Boer-Mulders</td>
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<td>density</td>
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<td>$f_{IT}^1$</td>
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<td>$h_{IT}$</td>
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<td>pretzelosity</td>
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</tbody>
</table>

### Diagram:

- **U**
  - unpolarized
- **T**
  - Collins

- FF
- DF
- e(E)
- e'($E'$)
- γ*
- h
- X
- $\sigma$

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Wednesday, August 31, 2011
1-hadron production x-section

\[ d\sigma = d\sigma^0_{UU} + \cos(2\phi) d\sigma^1_{UU} + \frac{1}{Q} \cos(\phi) d\sigma^2_{UU} + P_l \frac{1}{Q} \sin(\phi) d\sigma^3_{LU} \]

\[ + S_L \left[ \sin(2\phi) d\sigma^4_{UL} + \frac{1}{Q} \sin(\phi) d\sigma^5_{UL} + P_l \left( d\sigma^6_{LL} + \frac{1}{Q} \cos(\phi) d\sigma^7_{LL} \right) \right] \]

\[ + S_T \left[ \sin(\phi - \phi_s) d\sigma^8_{UT} + \sin(\phi + \phi_s) d\sigma^9_{UT} + \sin(3\phi - \phi_s) d\sigma^{10}_{UT} + \frac{1}{Q} \sin(2\phi - \phi_s) d\sigma^{11}_{UT} + \frac{1}{Q} \sin(\phi_s) d\sigma^{12}_{UT} \right] \]

\[ P_l \left[ \cos(\phi - \phi_s) d\sigma^{13}_{LT} + \frac{1}{Q} \cos(\phi_s) d\sigma^{14}_{LT} + \frac{1}{Q} \cos(2\phi - \phi_s) d\sigma^{15}_{LT} \right] \]
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\[ P_l \left( \cos(\phi - \phi_s) d\sigma_{LT}^{13} + \frac{1}{Q} \cos(\phi_s) d\sigma_{LT}^{14} + \frac{1}{Q} \cos(2\phi - \phi_s) d\sigma_{LT}^{15} \right) \]
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\]
1-hadron production $x$-section

$$d\sigma = d\sigma_{UU}^0 + \cos(2\phi) d\sigma_{UU}^1 + \frac{1}{Q} \cos(\phi) d\sigma_{UU}^2 + P_l \frac{1}{Q} \sin(\phi) d\sigma_{LU}^3$$

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$$\left. + P_l \left( \cos(\phi - \phi_s) d\sigma_{LT}^{13} + \frac{1}{Q} \cos(\phi_s) d\sigma_{LT}^{14} + \frac{1}{Q} \cos(2\phi - \phi_s) d\sigma_{LT}^{15} \right) \right]$$

disentangling the contributions:
- experiments with beam and target polarization states (U, L, T)
- extract the relevant Fourier amplitudes based on their azimuthal dependences

$$N(\phi, \phi_s) = \sigma_{UU}^0 \left\{ 1 + 2\langle \cos \phi \rangle_{UU} \cos \phi + 2\langle \cos 2\phi \rangle_{UU} \cos 2\phi \right.$$ 

$$+ S_T \left( 2\langle \sin(\phi - \phi_s) \rangle_{UT} \sin(\phi - \phi_s) + 2\langle \sin(\phi + \phi_s) \rangle_{UT} \sin(\phi + \phi_s) + 

2\langle \sin(3\phi - \phi_s) \rangle_{UT} \sin(\phi + \phi_s) + \ldots \right) \right.$$ 

$$+ S_T P_l \left( 2\langle \cos(\phi - \phi_s) \rangle_{UT} \cos(\phi - \phi_s) + 2\langle \cos \phi_s \rangle_{UT} \cos \phi_s + 

2\langle \cos(2\phi - \phi_s) \rangle_{UT} \cos(\phi - \phi_s) \right) \right\}$$

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\[\text{experiments with beam and target polarization states (U, L, T)}\]

\[\text{extract the relevant Fourier amplitudes based on their azimuthal dependences}\]

\[\text{if no perfect detection efficiency:}\]

$$N(\phi, \phi_s) = \epsilon(\phi, \phi_s) \sigma_{UU}^0 \left\{ 1 + 2\langle\cos\phi\rangle_{UU} \cos\phi + 2\langle\cos 2\phi\rangle_{UU} \cos 2\phi \right. +$$

$$S_T \left( 2\langle\sin(\phi - \phi_s)\rangle_{UT} \sin(\phi - \phi_s) + 2\langle\sin(\phi + \phi_s)\rangle_{UT} \sin(\phi + \phi_s) +$$

$$2\langle\sin(3\phi - \phi_s)\rangle_{UT} \sin(\phi + \phi_s) + \ldots \right)$$

$$+ S_T P_l \left( 2\langle\cos(\phi - \phi_s)\rangle_{UT} \cos(\phi - \phi_s) + 2\langle\cos \phi_s\rangle_{UT} \cos(\phi_s +$$

$$2\langle\cos(2\phi - \phi_s)\rangle_{UT} \cos(\phi - \phi_s) \right\}$$

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**disentangling the contributions:**

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**fit the cross section asymmetry for opposite spin states**

- systematics of neglecting cosine terms found to be negligible

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leading twist amplitudes
Collins effect

\[
d\sigma = d\sigma_{UU}^0 + \cos(2\phi)d\sigma_{UU}^1 + \frac{1}{Q}\cos(\phi)d\sigma_{UU}^2 + P_l \frac{1}{Q}\sin(\phi)d\sigma_{LU}^3 \\
+ S_L \left[ \sin(2\phi)d\sigma_{UL}^4 + \frac{1}{Q}\sin(\phi)d\sigma_{UL}^5 + P_l \left( d\sigma_{LL}^6 + \frac{1}{Q}\cos(\phi)d\sigma_{LL}^7 \right) \right] \\
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P_l \left( \cos(\phi - \phi_s)d\sigma_{LT}^{13} + \frac{1}{Q}\cos(\phi_s)d\sigma_{LT}^{14} + \frac{1}{Q}\cos(2\phi - \phi_s)d\sigma_{LT}^{15} \right)
\]

"Collins-effect" accounts for the correlation between the transverse spin of the fragmenting quark and the transverse momentum of the produced unpolarized hadron.

The transversity DF \( h_1^q(x) \) is sensitive to the difference of the number densities of transversely polarized quarks aligned along or opposite to the polarization of the nucleon.

Generates left-right (azimuthal) asymmetries.

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non-zero Collins effect observed!
both Collins FF and transversity sizable
non-zero Collins effect observed!
both Collins FF and transversity sizable

positive amplitude for $\pi^+$
compatible with zero amplitude for $\pi^0$
large negative amplitude for $\pi^-$
increase in magnitude with $x$
transversity mainly receives contribution from valence quarks
increase with $z$
in qualitative agreement with BELLE results

$2\langle \sin(\phi + \phi_s) \rangle_{UT} \propto \frac{C[-\hat{P}_h \cdot k_T h_1^q(x, p_T^2) H_{1}^{q \rightarrow h}(z, k_T^2)]}{C[f_1^q(x, p_T^2) D_1^{q \rightarrow h}(z, k_T^2)]}$
non-zero Collins effect observed!
both Collins FF and transversity sizable

positive amplitude for $\pi^+$
compatible with zero amplitude for $\pi^0$
large negative amplitude for $\pi^-$
increase in magnitude with $x$
transversity mainly receives contribution from valence quarks
increase with $z$
in qualitative agreement with BELLE results
positive for $\pi^+$ and negative for $\pi^-$

role of disfavored Collins FF:

$H_{1,\text{disf}} \approx -H_{1,\text{fav}}$

$u \Rightarrow \pi^+$; $d \Rightarrow \pi^-(\text{fav})$

$u \Rightarrow \pi^-$; $d \Rightarrow \pi^+(\text{disf})$

$h_1^u > 0$

$h_1^d < 0$

Cowins amplitudes for pions

2\left(\sin(\phi + \phi_s)\right)_{UT} \propto \frac{C \left[ -\hat{P}_{h^+}^\perp \cdot k_T^+ h_1^q(x, p_T^2) H_1^{q \rightarrow h}(z, k_T^2) \right]}{C \left[ f_1^q(x, p_T^2) D_1^{q \rightarrow h}(z, k_T^2) \right]}
Collins amplitudes for kaons

Collins amplitudes for kaons are similar to π⁺ as expected from the u-quark dominance.

\[ 2 \langle \sin(\phi + \phi_s) \rangle_{UT} \propto \frac{C \left[ -\frac{P_{h\perp} \cdot k_T}{M_h} h_1^q(x, p_{T}^2) H_{1}^{q\rightarrow h}(z, k_{T}^2) \right]}{C \left[ f_1^q(x, p_{T}^2) D_{1}^{q\rightarrow h}(z, k_{T}^2) \right]} \]

**K⁺**
- Similar to π⁺
- Consistent with u-quark dominance

**K⁻**
- Consistent with zero amplitudes
- K⁻ (ūs) is all see object
Collins amplitudes for kaons

\[ 2\langle \sin(\phi + \phi_s) \rangle_{UT} \propto \frac{C[-\frac{p_{h\perp} \cdot k_T}{M_h} h_1^q(x, p_T^2) H_{1q}^{\rightarrow h}(z, k_T^2)]}{C[f_1^q(x, p_T^2) D_{1q}^{\rightarrow h}(z, k_T^2)]} \]

**K**
- **K**\(^+\) amplitudes are similar to \(\pi^+\) as expected from the u-quark dominance
- **K**\(^+\) are larger than \(\pi^+\)

**K**
- consistent with zero amplitudes
- \(K^- (\bar{u}s)\) is all see object
Collins amplitudes for kaons

\[ 2\langle \sin(\phi + \phi_s) \rangle_{UT} \propto \frac{C \left[ -\frac{P_{h_\perp} \cdot k_T}{M_h} h_1^q(x, p_T^2) H_1^q \rightarrow h(z, k_T^2) \right]}{C \left[ f_1^q(x, p_T^2) D_1^q \rightarrow h(z, k_T^2) \right]} \]

\( K^+ \)
- \( K^+ \) amplitudes are similar to \( \pi^+ \) as expected from the u-quark dominance
- \( K^+ \) are larger than \( \pi^+ \)

\( K^- \)
- consistent with zero amplitudes
- \( K^- (\bar{u}s) \) is all see object

differences between \( K^+ \) and \( \pi^+ \) amplitudes
- role of sea quarks in conjunction with possibly large FF
- various contributions from decay of semi-inclusively produced vector-mesons
- the \( k_T \) dependences of the fragmentation functions

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\[
d\sigma = \sigma^0_{UU} + \cos(2\phi)d\sigma^1_{UU} + \frac{1}{Q}\cos(\phi)d\sigma^2_{UU} + P_L\frac{1}{Q}\sin(\phi)d\sigma^3_{LU} \\
+ S_L\left[\sin(2\phi)d\sigma^4_{UL} + \frac{1}{Q}\sin(\phi)d\sigma^5_{UL} + P_L\left(d\sigma^6_{LL} + \frac{1}{Q}\cos(\phi)d\sigma^7_{LL}\right)\right] \\
+ S_T\left[\sin(\phi - \phi_s)d\sigma^8_{UT} + \sin(\phi + \phi_s)d\sigma^9_{UT} + \sin(3\phi - \phi_s)d\sigma^{10}_{UT} + \frac{1}{Q}\sin(2\phi - \phi_s)d\sigma^{11}_{UT} + \frac{1}{Q}\sin(\phi_s)d\sigma^{12}_{UT}\right] \\
+ P_L\left(\cos(\phi - \phi_s)d\sigma^{13}_{LT} + \frac{1}{Q}\cos(\phi_s)d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi - \phi_s)d\sigma^{15}_{LT}\right)
\]

Sivers distribution function \( f_{1T}^{q}(x, p_T^2) \) describes the probability to find an unpolarized quark in a transversely polarized nucleon; gives the correlation between parton transverse momentum and transverse spin of the nucleon

- non-zero Sivers function implies non-zero orbital angular momentum
- correspondence between TMDs and GPDs: Sivers function and GPD E
- due to the final state interactions, Sivers effect generates left-right (azimuthal) asymmetries
Sivers amplitudes for pions

\[ 2\langle \sin(\phi - \phi_s)\rangle_{\text{UT}} = -\frac{\sum_q e_q^2 f_{1T}^q(x, p_T^2) \otimes w D_1^q(z, k_T^2)}{\sum_q e_q^2 f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)} \]

\[ \pi^+ \]
- significantly positive
- clear rise with \( z \)
- rise at low \( P_{h\perp} \), plateau at high \( P_{h\perp} \)
- dominated by scattering off u-quark:

\[ \simeq -\frac{f_{1T}^u(x, p_T^2) \otimes w D_1^{u\to\pi^+}(z, k_T^2)}{f_1^u(x, p_T^2) \otimes D_1^{u\to\pi^+}(z, k_T^2)} \]
- u-quark Sivers DF<0
- non-zero orbital angular momentum

\[ \pi^0 \]
- slightly positive

\[ \pi^- \]
- consistent with 0
- u- and d-quark cancellation
- d-quark Sivers DF>0
the pion difference asymmetry

\[ A_{\pi^+\pi^-}^{UT} = \frac{1}{\langle |S_T| \rangle} \frac{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) - (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})}{(\sigma_{U\uparrow}^{\pi^+} - \sigma_{U\uparrow}^{\pi^-}) + (\sigma_{U\downarrow}^{\pi^+} - \sigma_{U\downarrow}^{\pi^-})} \]

non-negligible contribution from exclusive \( \rho^0 \) largely cancels out

significantly positive Sivers amplitudes

\[ \langle \sin(\phi - \phi_s) \rangle_{UT}^{\pi^+\pi^-} \approx -\frac{4f_{1T}^u_{\nu} - f_{1T}^{d\nu}}{4f_{1T}^{u\nu} - f_{1T}^{d\nu}} \]

provides access to Sivers u-valence distribution

either \( f_{1T}^{d\nu} \gg f_{1T}^u_{\nu} \)

or \( f_{1T}^u_{\nu} \) is large and negative

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Sivers amplitudes for kaons

\[ 2 \langle \sin(\phi_s) \rangle_{ut} \]

\[ K^+ \]
- significantly positive
- clear rise with \( z \)
- rise at low \( P_{h\perp} \), plateau at high \( P_{h\perp} \)

\[ K^- \]
- slightly positive

\[ \sin(\phi_s) \]

\[ UT \]

\[ K^+ \]

\[ K^- \]
Sivers amplitudes for kaons

$K^+$ significantly positive
- clear rise with $z$
- rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$

$K^-$ slightly positive
Sivers amplitudes for kaons

- **$K^+$**: significantly positive
  - clear rise with $z$
  - rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$

- **$K^-$**: slightly positive
  - similar to $\pi^+$, $K^+$ dominated by scattering off $u$-quarks:

$$\alpha \propto f_{1T}^u(x, p_T^2) \otimes w D_1^{u \to \pi^+/K^+}(z, k_T^2)$$

---

*Sivers amplitudes for kaons*
Sivers amplitudes for kaons

$K^+$
- significantly positive
- clear rise with $z$
- rise at low $P_{h\perp}$, plateau at high $P_{h\perp}$

$K^-$
- slightly positive

similar to $\pi^+$, $K^+$ dominated by scattering off u-quarks:

$$K^+ \equiv \begin{bmatrix} u \bar{d} \end{bmatrix} \quad \pi^+ \equiv \begin{bmatrix} u \bar{d} \end{bmatrix}$$

non-trivial role of sea quarks

different $k_T$ dependence of fragmentation functions

higher-twist effects

$\alpha = \frac{f_{1T,u}(x, p_T^2)}{f_{1T,u}(x, p_T^2)} \otimes D_{u \to \pi^+/K^+}^{u}(z, k_T^2)$

K+ amplitudes are larger in size than the $\pi^+$ amplitudes
“Pretzelosity”

\[ d\sigma = d\sigma^0_{UU} + \cos(2\phi)d\sigma^1_{UU} + \frac{1}{Q}\cos(\phi)d\sigma^2_{UU} + P_l\frac{1}{Q}\sin(\phi)d\sigma^3_{LU} \]
\[ + S_L\left[\sin(2\phi)d\sigma^4_{UL} + \frac{1}{Q}\sin(\phi)d\sigma^5_{UL} + P_l\left(d\sigma^6_{LL} + \frac{1}{Q}\cos(\phi)d\sigma^7_{LL}\right)\right] \]
\[ + S_T\left[\sin(\phi - \phi_s)d\sigma^8_{UT} + \sin(\phi + \phi_s)d\sigma^9_{UT} + \sin(3\phi - \phi_s)d\sigma^{10}_{UT} - \frac{1}{Q}\sin(2\phi - \phi_s)d\sigma^{11}_{UT} + \frac{1}{Q}\sin(\phi_s)d\sigma^{12}_{UT}\right] \]
\[ + P_l\left(\cos(\phi - \phi_s)d\sigma^{13}_{LT} + \frac{1}{Q}\cos(\phi_s)d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi - \phi_s)d\sigma^{15}_{LT}\right) \]

“pretzelosity” DF \( h_{1T}^q(x, p_T^2) \) gives a measure of the deviation of the nucleon shape from a sphere

- correlation between parton transverse momentum and parton transverse polarization in a transversely polarized nucleon
- it is expected to be suppressed at small and large \( x \) w.r.t. \( f_1^q, g_1^q, h_1^q \)
- satisfies the positivity condition: \( h_{1T}^q \leq \frac{1}{2}(f_1^q + g_1^q) \)
- envolve quark and nucleon helicity flips; is related to chiral-odd GPD
- gives the measure of ‘relativistic effects’ in the nucleon: \( \frac{p_T^2}{2M^2} h_{1T}^q(x, p_T^2) = g_1^q(x, p_T^2) - h_1^q(x, p_T^2) \)

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$\sin(3 \phi - \phi_s)$ amplitudes

$$2 \langle \sin(3\phi - \phi_s) \rangle_{UT} \propto \frac{\sum_q e_q^2 x h_1^\perp (1), q(x) \otimes_w H_1^\perp (1/2) q(z)}{\sum_q e_q^2 f_1^q(x) \otimes D_1^q(z)}$$

suppressed by two powers of $P_{h\perp}$ compared to Collins and Sivers amplitudes

compatible with zero within uncertainties

pretzelosity might be non-zero at higher $P_{h\perp}$
\[
\begin{align*}
\frac{d\sigma}{dQ} &= \frac{d\sigma^0_{UU}}{dQ} + \cos(2\phi)\frac{d\sigma^1_{UU}}{dQ} + \frac{1}{Q} \cos(\phi)\frac{d\sigma^2_{UU}}{dQ} + P_l \frac{1}{Q} \sin(\phi)\frac{d\sigma^3_{LU}}{dQ} \\
+ &\ S_L \left[ \sin(2\phi)\frac{d\sigma^4_{UL}}{dQ} + \frac{1}{Q} \sin(\phi)\frac{d\sigma^5_{UL}}{dQ} + P_l \left( \frac{1}{Q} \sin(\phi)\frac{d\sigma^6_{LL}}{dQ} + \frac{1}{Q} \cos(\phi)\frac{d\sigma^7_{LL}}{dQ} \right) \right] \\
+ &\ S_T \left[ \sin(\phi - \phi_s)\frac{d\sigma^8_{UT}}{dQ} + \sin(\phi + \phi_s)\frac{d\sigma^9_{UT}}{dQ} + \sin(3\phi - \phi_s)\frac{d\sigma^{10}_{UT}}{dQ} + \frac{1}{Q} \sin(2\phi - \phi_s)\frac{d\sigma^{11}_{UT}}{dQ} + \frac{1}{Q} \sin(\phi_s)\frac{d\sigma^{12}_{UT}}{dQ} \right] \\
+ &\ P_l \left[ \cos(\phi - \phi_s)\frac{d\sigma^{13}_{LT}}{dQ} + \frac{1}{Q} \cos(\phi_s)\frac{d\sigma^{14}_{LT}}{dQ} + \frac{1}{Q} \cos(2\phi - \phi_s)\frac{d\sigma^{15}_{LT}}{dQ} \right] 
\end{align*}
\]

Worm-gear DF $g_{1T}^q(x, p_T^2)$ and $h_{1L}^{1,q}(x, p_T^2)$ describes the probability to find a longitudinal/transversely polarized quark in a transversely/longitudinally polarized nucleon

- on a transversely target $h_{1L}^{1,q}(x, p_T^2)$ accessible in the measurements through $\sin(2\phi + \phi_s)$

Fourier component

- gives correlation between parton transverse momentum and parton longitudinal / transverse polarization in a longitudinal / transversely polarized nucleon

- model dependent relations:

\[
\begin{align*}
g_{1T}^{1,q}(x, p_T^2) &\approx x \int_x^1 \frac{1}{y} g_1^q(y, p_T^2) dy \\
h_{1L}^{1,q}(x, p_T^2) &=-g_{1T}^{1,q}(x, p_T^2) \\
h_{1L}^{1,q}(x, p_T^2) &\approx -x \int_x^1 \frac{1}{y} h_1^q(y, p_T^2) dy
\end{align*}
\]

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The cos(\(\phi - \phi_s\)) amplitudes

\[
2\langle\cos(\phi - \phi_s)\rangle_{LT} \propto \frac{C \left[ -\frac{\hat{P}_{h+} \cdot p_T}{M_h} g_{1T}^q(x, p_T^2) D_{1}^q(z, k_T^2) \right]}{C \left[ f_1^q(x, p_T^2) D_{1}^q(z, k_T^2) \right]}
\]

- Uncertainties are larger than in single-spin asymmetries scaled by the beam polarization value.
- \(\pi^+\) slightly positive
- \(\pi^0\) compatible with zero
- \(\pi^-\) positive
  - Evidence for non-zero worm-gear distribution
- \(K^+\) slightly positive
- \(K^-\) compatible with zero

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subleading-twist amplitudes
the subleading-twist \( \sin(2 \phi + \phi_s) \) amplitudes

 arises solely from longitudinal component of the target spin:

\[
P_{T \perp} A_{U\perp}(\phi, \phi_s) = S_{T \perp} A_{UT}(\phi, \phi_s) + S_L A_{UL}
\]

- longitudinal component of the target spin <15\%
- expected to scale as \( \sin \theta_{y^*} \langle \sin(2\phi) U_L \rangle \)
- related to worm-gear DF \( h_{11L} \)
- \( \sin(2\phi + \phi_s) \) amplitude is suppressed by one powers of \( P_{h\perp} \) compared to Collins and Sivers amplitudes
- compatible with zero within uncertainties except maybe K⁺
\[ d\sigma = d\sigma^0_{UU} + \cos(2\phi)d\sigma^1_{UU} + \frac{1}{Q} \cos(\phi)d\sigma^2_{UU} + P_l \frac{1}{Q} \sin(\phi)d\sigma^3_{LU} \]
\[ + S_L \left[ \sin(2\phi)d\sigma^4_{UL} + \frac{1}{Q} \sin(\phi)d\sigma^5_{UL} + P_l \left( d\sigma^6_{LL} + \frac{1}{Q} \cos(\phi)d\sigma^7_{LL} \right) \right] \]
\[ + S_T \left[ \sin(\phi - \phi_s)d\sigma^8_{UT} + \sin(\phi + \phi_s)d\sigma^9_{UT} + \sin(3\phi - \phi_s)d\sigma_{10}^{10} + \frac{1}{Q} \sin(2\phi - \phi_s)d\sigma_{11}^{11} + \frac{1}{Q} \sin(\phi_s)d\sigma_{12}^{12} \right] \]
\[ + P_l \left( \cos(\phi - \phi_s)d\sigma_{13}^{13} + \frac{1}{Q} \cos(\phi_s)d\sigma_{14}^{14} + \frac{1}{Q} \cos(2\phi - \phi_s)d\sigma_{15}^{15} \right) \]
survive the integration over $P_{h\perp}$

$$F_{UT}^{\sin \phi_s}(x, z, Q^2) = -x \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$

in one-photon approximation

$$\sum_z \int dz z F_{UT}^{\sin \phi_s}(x, z, Q^2) = 0$$

receives various contributions

$$\propto x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z}$$

$$-W_1(p_T, k_T) \left( x h_T H_1^\perp + \frac{M_h}{M} g_1 T \frac{\tilde{G}^\perp}{z} - x h_T^\perp H_1^\perp + \frac{M_h}{M} f_1^\perp \frac{\tilde{D}^\perp}{z} \right)$$

non-zero signal observed for $\pi^-$ and $K^-$
the subleading-twist
\( \sin(2 \phi - \phi_s) \)

\[
\propto W_1(p_T, k_T, P_{h\perp}) \left( x f_T^{\perp} D_1 - \frac{M_h}{M} h_1^T \frac{\tilde{H}}{z} \right) \\
-W_2(p_T, k_T, P_{h\perp}) \left( x h_T H_1^T + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^{\perp}}{z} + x h_T H_1^T - \frac{M_h}{M} f_{1T}^{\perp} \frac{\tilde{D}^{\perp}}{z} \right)
\]

- suppressed by two power of \( P_{h\perp} \) and an additional factor \( 2M/Q \) compared to Collins and Sivers amplitudes
- compatible with zero within uncertainties
compatible with zero subleading-twist
\( \cos \phi_s \) and \( \cos (2 \phi - \phi_s) \) amplitudes
ongoing
analysis with
higher statistical
precision

**Summary**

\[ d\sigma = d\sigma^0_{UU} + \cos(2\phi)d\sigma^1_{UU} + \frac{1}{Q}\cos(\phi)d\sigma^2_{UU} + P_t \left( \frac{1}{Q}\sin(\phi)d\sigma^3_{LU} \right) \]

\[ + S_L \left[ \sin(2\phi)d\sigma^4_{UL} + \frac{1}{Q}\sin(\phi)d\sigma^5_{UL} \right] + P_t \left( d\sigma^6_{LL} + \frac{1}{Q}\sin(\phi)d\sigma^7_{LL} \right) \]

\[ + S_T \left[ \sin(\phi - \phi_s)d\sigma^8_{UT} + \sin(\phi + \phi_s)d\sigma^9_{UT} + \sin(3\phi - \phi_s)d\sigma^{10}_{UT} + \frac{1}{Q}\sin(2\phi - \phi_s)d\sigma^{11}_{UT} + \frac{1}{Q}\sin(\phi_s)d\sigma^{12}_{UT} \right] \]

\[ + P_t \left( \cos(\phi - \phi_s)d\sigma^{13}_{LT} + \frac{1}{Q}\cos(\phi_s)d\sigma^{14}_{LT} + \frac{1}{Q}\cos(2\phi - \phi_s)d\sigma^{15}_{LT} \right) \]
TSA in inclusive hadron production in $p^\uparrow p$

Measurements of $A_N = \frac{N_R - N_L}{N_R + N_L}$ in $p^\uparrow p \rightarrow \pi X$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Energy (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANL (1976)</td>
<td>$\sqrt{s} = 4.9$ GeV</td>
</tr>
<tr>
<td>BNL (2002)</td>
<td>6.6 GeV</td>
</tr>
<tr>
<td>FNAL (1991)</td>
<td>19.4 GeV</td>
</tr>
<tr>
<td>RHIC (2008)</td>
<td>62.4 GeV</td>
</tr>
</tbody>
</table>

Interpretations:
- TMDs (Sivers effect)
- Twist-3 $qg$ correlators

Suggest:
- Increase of $A_N$ with increase of $x_F$
- Decrease of $A_N$ with increase of $p_T$ at fixed $x_F$
- $A_N \rightarrow 0$ at high $p_T$
inclusive hadron production

DIS variables: $Q^2, x$

inclusive hadron production: $x_F, P_T$

no scattered lepton detection

$A_{UT} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} = A_{UT}^{\sin \phi} \sin \phi$

$A_N = \frac{\int d\phi \sigma_{UT} \sin \phi}{\int d\phi \sigma_{UU}} = -\frac{2}{\pi} A_{UT}^{\sin \phi}$
\( A_{UT}^{\sin \phi} \% p_T & x_F \)

- \( \pi^+, K^+ \):
  - positive
- \( \pi^- \):
  - slightly negative
- \( K^- \):
  - compatible with zero

\( \pi^+ \) and \( K^+ \) asymmetries decrease at high \( p_T \)

---

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$A_{UT}^{\sin \phi}$ % $p_T$ & $x_F$

- HERMES preliminary

$e^\pm p^\uparrow \to \pi^\pm + X$

$e^\pm p^\uparrow \to \pi^* + X$

$e^\pm p^\uparrow \to K^\pm + X$

$0.20 < x_F < 1.00$

8.8% scale uncertainty

$0.08 < x_F < 0.20$

$-0.10 < x_F < 0.08$

sign change for $\pi^-$

positive $K^-$ for $x_F \approx 0$

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Wednesday, August 31, 2011
comparison to SIDIS measurements

\[ 2 \langle \sin(\phi - \phi_s) \rangle / \sigma \]

- \( \pi^+ \)
- \( \pi^0 \)
- \( \pi^- \)

\( p_T \) [GeV]

\( x \)

\( z \)

\( P_{h\perp} \) [GeV]
comparison to previous measurements

\[ e^\pm p^\uparrow \rightarrow \pi^\pm + X \text{ HERMES preliminary} \]

\( A_{UT}^{\sin} \)

\( A_{UT}^{\sin} \)

\( e^\pm p^\uparrow \rightarrow \pi^- + X \)

\( A_{UT}^{\sin} \)

\( \langle p_\uparrow \rangle [\text{GeV}] \)

\( x_F \)

\( \pm 0.8 \text{ scale uncertainty} \)

\( u\)-quark dominance in \( ep^\uparrow \) may explain the smaller size of \( \pi^- \) asymmetry

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backup slides
the subleading-twist $\sin \phi_s$ amplitudes

$\pi^-$ $\sin \phi_s$ amplitude and Collins amplitude

$\pi^-$ similarities in size and the shape

might be due to correlations between amplitudes

might be explained also by physics

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Wednesday, August 31, 2011