Are parton distribution functions universal?

Alessandro Bacchetta
Are parton distribution functions universal or not?

Alessandro Bacchetta
From March 2008

Nathan Isgur Fellow at

Jefferson Lab
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Jefferson Lab
Outline
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- Factorization and universality
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- $k_T$ factorization and unintegrated parton distribution functions
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- $k_T$ factorization and unintegrated parton distribution functions
- Gauge links in PDFs
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- Gauge links in PDFs
- Gauge links in different processes
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- Gauge links in different processes
- Problems with universality
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- Factorization and universality
- $k_T$ factorization and unintegrated parton distribution functions
- Gauge links in PDFs
- Gauge links in different processes
- Problems with universality
- Conclusions
DIS

\[ \ell + p \rightarrow \ell + X \]
**DIS**

\[ \ell + p \rightarrow \ell + X \]

**Drell-Yan**

\[ p + p \rightarrow \ell + \bar{\ell} + X \]
**DIS**

\[ \ell + p \rightarrow \ell + X \]

**Drell-Yan**

\[ p + p \rightarrow \ell + \overline{\ell} + X \]

**pp to hadrons**

\[ p + p \rightarrow h_1 + h_2 + X \]
**DIS**

\[ \ell + p \rightarrow \ell + X \]

**Drell-Yan**

\[ p + p \rightarrow \ell + \bar{\ell} + X \]

**pp to jets**

\[ p + p \rightarrow j_1 + j_2 + X \]
Factorization

**DIS**

lepton

proton
Factorization

**DIS**

- Lepton
- Proton
Factorization

DIS

lepton

proton
Factorization

DIS

lepton

proton

Partonic scattering amplitude

Distribution amplitude
Factorization

**DIS**

\[ d\sigma = H \otimes f \]

- **Partonic scattering amplitude**
- **Distribution amplitude**
Universality

\[ \ell + p \rightarrow \ell + X \]
Universality

**DIS**

\[ \ell + p \rightarrow \ell + X \]

**Drell-Yan**

\[ p + p \rightarrow \ell + \bar{\ell} + X \]
Universality

**DIS**

\[ \ell + p \rightarrow \ell + X \]

**Drell-Yan**

\[ p + p \rightarrow \ell + \bar{\ell} + X \]

**UNIVERSALITY**
Key concepts in QCD!

Factorization and universality are at the base of much of the predictive power of QCD. For instance, they give the possibility to extract PDFs from HERA data and use them to look for new physics at LHC.
Jet semi-inclusive DIS

\[ \ell(l) + p(P) \rightarrow \ell(l') + j(k_j) + X \]
Jet semi-inclusive DIS

$$\ell(l) + p(P) \rightarrow \ell(l') + j(k_j) + X$$

$$-(l - l')^2 = Q^2 = \text{virtuality of photon}$$

lepton $l$

proton

jet
Jet semi-inclusive DIS

\[ \ell(l) + p(P) \rightarrow \ell(l') + j(k_j) + X \]

\[ -(l - l')^2 = Q^2 = \text{virtuality of photon} \quad x = \frac{Q^2}{2P \cdot (l - l')} \]

lepton \[ l \]

jet

proton
Jet semi-inclusive DIS

\[ \ell(l) + p(P) \rightarrow \ell(l') + j(k_j) + X \]

\[-(l - l')^2 = Q^2 = \text{virtuality of photon}\]

\[ x = \frac{Q^2}{2P \cdot (l - l')} \]
Jet semi-inclusive DIS

\[ \ell(l) + p(P) \rightarrow \ell(l') + j(k_j) + X \]

\[ -(l - l')^2 = Q^2 = \text{virtuality of photon} \]

\[ x = \frac{Q^2}{2P \cdot (l - l')} \]
Transverse momentum effects

SIDIS

$k_T$
Transverse momentum effects

SIDIS

Drell-Yan

$k_T$

$q_T$
Transverse momentum effects

SIDIS

$e^-e^+ \to \text{pions}$

Drell-Yan

$k_T$

$q_T$

$K_T$
Transverse momentum effects

SIDIS

$e^{-}e^{+}$ to pions

$K_T$

Drell-Yan

$p$-$p$ to pions

$3$-$D$

$T_T$

$T_q$

$T_K$

$T_R$

$T_R$

$T_T$

$T_T$

$T_T$

$T_T$
Whenever we measure transverse-momentum effects, we need $k_T$-factorization and we need transverse momentum dependent (or unintegrated) parton distributions

Collins, Soper, NPB 193 (81)
$k_T$ factorization and universality

SIDIS
$k_T$ factorization and universality

SIDIS

Drell-Yan
$k_T$ factorization and universality

SIDIS

$e^-e^+ \to \text{pions}$

Drell-Yan
$k_T$ factorization and universality

SIDIS

Drell-Yan

UNIVERSALITY

e^-e^+ to pions
$k_T$ factorization and universality

SIDIS

$e^−e^+ \rightarrow \text{pions}$

Drell-Yan

$p+p \rightarrow \text{pions}$
$k_T$ factorization and universality

**SIDIS**
- Proton
- Lepton
- Pion

**Drell-Yan**
- Proton
- Lepton
- Antilepton

$e^-e^+ \text{ to pions}$

$p-p \text{ to pions}$
$k_T$ factorization and universality

SIDIS

$e^-e^+ \rightarrow$ pions

Drell-Yan

$p-p \rightarrow$ pions
Parton distribution functions
Parton distribution functions

\[
P, S \quad k = \Phi
\]
Parton distribution functions

\[ \Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left\langle P \left| \bar{\psi}_j(0) \psi_i(\xi^-) \right| P \right\rangle \]
Parton distribution functions

\[ \Phi_{ij}(x) = \int \frac{d\xi^-}{2\pi} e^{ix P^+ \xi^-} \left< P \left| \bar{\psi}_j(0) U_{[0,\xi^-]} \psi_i(\xi^-) \right| P \right> \]
Gauge link or Wilson line
Gauge link or Wilson line

\[ T \]

\[ \xi \]

\[ \mathcal{U}_{[0,\xi^{-}]} \]
Gauge link or Wilson line

At leading order in $1/Q$

\[ U_{[0, \xi^-]}^\prime \equiv \mathcal{P} \exp \left( -ig \int_0^{\xi^-} d\zeta^- A^+ \right) \]
Unintegrated parton distribution functions

\[ k \quad 2 \quad \Phi \]
Unintegrated parton distribution functions

$$\Phi_{ij}(x, k_T) = \int \frac{d\xi^- d^2 \xi_T}{8\pi^3} e^{ik \cdot \xi} \left< P \left| \psi_j(0) U_{[0,\xi]} \psi_i(\xi) \right| P \right|_{\xi^+ = 0}$$
Gauge link

\[
\begin{align*}
T & \\
\xi_T & \\
\xi^{-} & 
\end{align*}
\]
Gauge link

\[ T \]

\[ \xi \]

\[ \xi_T \]

\[ \xi^- \]

\[ U^-_{[0,\xi_T]} \]

\[ U^T_{[0,\xi_T]} \]

\[ U^-_{[\xi_T,\infty]} \]

\[ U^-_{[\infty,\xi^-]} \]
Gauge link

\[
\mathcal{U}^{-}_{[0,\infty]} \mathcal{U}^{T}_{[0,\xi_{T}]} \mathcal{U}^{-}_{[\infty,\xi^{-}]} \quad \text{at} \quad \infty^{-}
\]
Gauge link

\[
\mathcal{U}^{-}_{[0,\xi^{-}]} \equiv \mathcal{P} \exp \left( -ig \int_{0}^{\xi^{-}} d\zeta^{-} A_{+}^{-} \right) \quad \mathcal{U}^{T}_{[0,\xi_{T}]} \equiv \mathcal{P} \exp \left( -ig \int_{0}^{\xi_{T}} d\zeta_{T} A_{T} \right)
\]

at \( \infty^{-} \)
Obtaining the gauge link

\[
\begin{align*}
k + q - l & \quad k + q \\
k - l & \quad l
\end{align*}
\]

Ji, Yuan, PLB 543 (02)
Belitsky, Ji, Yuan, NPB656 (03)
Obtaining the gauge link

\[ k + q - l \]

\[ k - l \]

\[ k + q \]

\[ k + q \]

Ji, Yuan, PLB 543 (02)
Belitsky, Ji, Yuan, NPB656 (03)
Obtaining the gauge link

\[(K + q') g A \frac{(K + q' - l)}{(k + q - l)^2 + i \varepsilon} \gamma^\mu\]

\[k + q\]

\[k + q - l\]

\[k - l\]

\[\Phi\]

Ji, Yuan, PLB 543 (02)
Belitsky, Ji, Yuan, NPB656 (03)
Obtaining the gauge link

\[(k + q') g A \left( \frac{(k + q' - l)}{(k + q - l)^2 + i \epsilon} \right) \gamma^\mu\]

eikonal approximation: \((k + q) \approx q^- n_+\)

Ji, Yuan, PLB 543 (02)
Belitsky, Ji, Yuan, NPB656 (03)
Obtaining the gauge link

\[ (k + q) g A \left\{ \frac{(k + q - l)}{(k + q - l)^2 + i\varepsilon} \right\}^\mu \]

eikonal approximation: \( (k + q) \approx q^- n_+ \)

\[ \approx q^- \gamma^+ g A^+ \gamma^- \left\{ \frac{q^- \gamma^+}{-2q^- l^+ + i\varepsilon} \right\}^\mu \]

\[ = -q^- \gamma^+ \frac{\gamma^- \gamma^+}{2} \gamma^\mu g A^+ \left\{ \frac{1}{l^+ - i\varepsilon} \right\} \]

\[ \approx (k + q) \gamma^\mu \frac{g A^+}{-l^+ + i\varepsilon} \]

Ji, Yuan, PLB 543 (02)
Belitsky, Ji, Yuan, NPB656 (03)
Eikonal propagator

\[ (K + q)\gamma^\mu \frac{gA^+}{-l^+ + i\epsilon} \]

\[ P \]

\[ k + q \]

\[ k - l \]

\[ l \]

\[ eikonal propagator \]

\[ eikonal vertex \]
Eikonal propagator

\[\Phi_k - l + \Phi_{k+q}\]

\[(K + q)\gamma^\mu\]

\[\frac{gA^+}{-l^+ + i\epsilon}\]

\[\frac{gA^+}{-l^+ + i\epsilon} = -\text{PV} \frac{gA^+}{l^+} - i\pi \delta(l^+)gA^+\]
Gauge link at order $g$
Gauge link at order $g$

\[
\int \frac{d^4 l}{(2\pi)^4} \frac{gA^+(l)}{-l^+ + i\varepsilon} = -ig \int \frac{d^2 l_\perp d l^-}{(2\pi)^3} A^+(l)|_{l^+ = 0}
\]

\[
= -ig \int_0^\infty d\zeta^- A^+(\zeta)|_{\zeta^+ = \zeta_\perp = 0}
\]
Gauge link at order $g$

\[
\int \frac{d^4 l}{(2\pi)^4} \frac{gA^+(l)}{-l^+ + i\varepsilon} = -ig \int \frac{d^2 l_\perp dl^-}{(2\pi)^3} A^+(l)|_{l^+ = 0} \\
= -ig \int_0^\infty d\zeta^- A^+(\zeta)|_{\zeta^+ = \zeta_\perp = 0}
\]

$O(g)$ contribution to gauge link!

\[
\mathcal{U}_{[0,\infty]}^{-} \equiv \mathcal{P} \exp \left( -ig \int_0^\infty d\zeta^- A^+(\zeta^-) \right)
\]
Gauge link in different gauges

Feynman gauge  Axial gauge (A⁺=0)

\[ \xi^T \]

\[ \xi^\perp \]
Gauge link in different gauges

**Feynman gauge**

**Axial gauge (A⁺=0)**

\[ \mathcal{U}_{\infty,\xi^-_{\infty}} \]

\[ \mathcal{U}_{0,\xi^-} \]

\[ \mathcal{U}_{[0,\xi^-]}^{\infty} \equiv \mathcal{P} \exp \left( -ig \int_0^{\xi^-} d\xi^- A^+ \right) \]
Gauge link in different gauges

Feynman gauge

Axial gauge ($A^+ = 0$)

\[
\mathcal{U}^-_{[0, \xi^-]} \equiv \mathcal{P} \exp \left( -ig \int_0^{\xi^-} d\zeta^- A^+ \right)
\]

\[
\mathcal{U}^-_{[0, \xi_T]} \equiv \mathcal{P} \exp \left( -ig \int_0^{\xi_T} d\zeta_T \cdot A_T \right)
\]
Drell-Yan processes

Collins, PLB 536 (02)
Drell-Yan processes

\[ (q - K)gA \frac{(K - q - l)}{(k - q - l)^2 + i\epsilon} \gamma^\mu \]

\[ \approx q^- \gamma^+ gA^+ \gamma^- \frac{-q^- \gamma^+}{2q^- l^+ + i\epsilon} \gamma^\mu \]

\[ = q^- \gamma^+ \frac{\gamma^- \gamma^+}{2} \gamma^\mu \frac{gA^+}{-l^+ - i\epsilon} \]

\[ \approx (q - K) \gamma^\mu \frac{gA^+}{-l^+ - i\epsilon} \]

Collins, PLB 536 (02)
There is a change of sign in the imaginary part of the eikonal propagator.
Gauge link or Wilson line

\[
\mathcal{U}^{-}_{[0, \infty]} \mathcal{U}^{T}_{[0, \xi_T]} \mathcal{U}^{-}_{[\infty, \xi^{-}]} 
\]
Gauge link or Wilson line

**DIS**

\[ U^-_{[0, \infty]} \ U^T_{[0, \xi_T]} \ U^-_{[\infty, \xi^-]} \]

**Drell-Yan**

\[ U^-_{[0, -\infty]} \ U^T_{[0, \xi_T]} \ U^-_{[-\infty, \xi^-]} \]
Gauge link or Wilson line

**DIS**

\[ U_{[0, \infty]}^{-} U_{[0, \xi]}^{T} U_{[\infty, \xi^{-}]}^{-} \]

**Drell-Yan**

\[ U_{[0, -\infty]}^{-} U_{[0, \xi]}^{T} U_{[-\infty, \xi^{-}]}^{-} \]
Consequences
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- The real part of the gauge link remains unchanged
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- The imaginary part changes sign
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\[ d\sigma_{\text{DIS}} = H_{\text{DIS}} \otimes f \]
Consequences

- The real part of the gauge link remains unchanged.
- The imaginary part changes sign.
- Observables sensitive to the imaginary part (e.g., single spin asymmetries) acquire an extra minus sign (generalization of universality).

\[ d\sigma_{\text{DIS}} = H_{\text{DIS}} \otimes f \quad \text{and} \quad d\sigma_{\text{D-Y}} = H_{\text{D-Y}} \otimes f \]
Consequences

- The real part of the gauge link remains unchanged
- The imaginary part changes sign
- Observables sensitive to the imaginary part (e.g. single spin asymmetries) acquire an extra minus sign (generalization of universality)

\[
d\sigma_{\text{DIS}} = H_{\text{DIS}} \otimes f \quad \quad d\sigma_{\text{D-Y}} = H_{\text{D-Y}} \otimes f
\]

\[
d\sigma_{\text{DIS}}^{\uparrow} - d\sigma_{\text{DIS}}^{\downarrow} = K_{\text{DIS}} \otimes g
\]
Consequences

- The real part of the gauge link remains unchanged
- The imaginary part changes sign
- Observables sensitive to the imaginary part (e.g. single spin asymmetries) acquire an extra minus sign (generalization of universality)

\[
d\sigma_{DIS} = H_{DIS} \otimes f \\
d\sigma_{D-Y} = H_{D-Y} \otimes f \\
d\sigma_{DIS}^{\uparrow} - d\sigma_{DIS}^{\downarrow} = K_{DIS} \otimes g \\
d\sigma_{D-Y}^{\uparrow} - d\sigma_{D-Y}^{\downarrow} = -K_{D-Y} \otimes g
\]
A slightly more complex example

Collins, Qiu, PRD 75 (07)

\[ g_1 \left[ -l^+ + i\varepsilon \right] \]

parton with charge $g_1$
A slightly more complex example

\[ q' \]

\[ q \]

\[ \hat{\Phi} \]

\[ \Phi \]

\[ g_1 \left[ -l^+ + i\epsilon \right] \]

\[ g_2 \left[ -l^+ + i\epsilon \right] \]

Collins, Qiu, PRD 75 (07)
A slightly more complex example

Collins, Qiu, PRD 75 (07)

\[ \begin{align*} 
\text{parton with charge } g_2 & \\
\text{parton with charge } g_1 & \\
\end{align*} \]
Consequences

\[
\frac{g_1}{-l^+ + i\epsilon} + \frac{g_2}{-l^+ + i\epsilon} - \frac{g_2}{-l^+ - i\epsilon}
\]

\[
= -i\pi (2g_2 + g_1) \delta(l^+) - \text{PV} \frac{g_1}{l^+}
\]
Consequences

\[
g_1 \left[ -l^+ + i \epsilon \right] + g_2 \left[ -l^+ + i \epsilon \right] - g_2 \left[ -l^+ - i \epsilon \right] = -i \pi (2g_2 + g_1) \delta(l^+) - \text{PV} \frac{g_1}{l^+}
\]

- Up to this order, the real part is unchanged, the imaginary part gets more than just a simple sign change and depends on the charge of ANOTHER parton!
Consequences

\[
\frac{g_1}{-l^+ + i\epsilon} + \frac{g_2}{-l^+ + i\epsilon} - \frac{g_2}{-l^+ - i\epsilon}
\]

\[
= -i\pi (2g_2 + g_1) \delta(l^+) - \text{PV} \frac{g_1}{l^+}
\]

- Up to this order, the real part is unchanged, the imaginary part gets more than just a simple sign change and depends on the charge of ANOTHER parton!
- Still possible to get around it: PDFs could still be universal, but the ones sensitive to the imaginary part (those involved in single spin asymmetries) have to be multiplied by \( g_1/(2g_2+g_1) \)
Two-gluon exchange


+ more
Two-gluon exchange


\[
g_1^2 \left[ \frac{1}{k_1^+ k_2^+} + (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) \right] + g_1(g_1 + 2g_2)(i\pi) \left[ \frac{\delta(k_2^+)}{k_1^+} + \frac{\delta(k_1^+)}{k_2^+} \right] \\
+ 4 \left( g_1 g_2 + g_2^2 \right) (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) .
\]
Two-gluon exchange


\[
\begin{align*}
g_1^2 \left[ \frac{1}{k_1^+ k_2^+} + (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) \right] & + 4 \left( g_1 g_2 + g_2^2 \right) (-i\pi)^2 \delta(k_1^+) \delta(k_2^+) \\
& \text{Breaking of universality, and not only in single-spin asymmetries}
\end{align*}
\]

+ more
Infinitely many gluons and different processes

Bomhof, Mulders, Pijlman, PLB 596 (04)
Weighted cross sections

A.B., Bomhof, Mulders, Pijlman, PRD72 (05)
Weighted cross sections

\[ \int \frac{d\sigma_{DIS}}{dk_T} \, dk_T = H_{DIS} \otimes f \]
Weighted cross sections

\[ \int \frac{d\sigma_{\text{DIS}}}{dk_T} \, dk_T = H_{\text{DIS}} \otimes f \]

\[ \int \frac{d\sigma_{pp}}{dk_T} \, dk_T = H_{pp} \otimes f \]

A.B., Bomhof, Mulders, Pijlman, PRD72 (05)
Weighted cross sections

\[ \int \frac{d\sigma_{DIS}}{dk_T} \, dk_T = H_{DIS} \otimes f \]

\[ \int \frac{d\sigma_{pp}}{dk_T} \, dk_T = H_{pp} \otimes f \]

\[ \int (k_T \cdot n) \frac{d\sigma_{DIS}}{dk_T} \, dk_T = K_{DIS} \otimes g \]
Weighted cross sections

A.B., Bomhof, Mulders, Pijlman, PRD72 (05)

\[ \int \frac{d\sigma_{\text{DIS}}}{dk_T} \, dk_T = H_{\text{DIS}} \otimes f \]

\[ \int \frac{d\sigma_{pp}}{dk_T} \, dk_T = H_{pp} \otimes f \]

\[ \int (k_T \cdot n) \frac{d\sigma_{\text{DIS}}}{dk_T} \, dk_T = K_{\text{DIS}} \otimes g \]

\[ \int (k_T \cdot n) \frac{d\sigma_{pp}}{dk_T} \, dk_T = K_{pp} \otimes g' = K_{pp} \otimes C \, g \]

\[ = C \, K_{pp} \otimes g = K'_{pp} \otimes g \]
Example of phenomenology

A.B., D’Alesio, Bomhof, Mulders, Murgia, PRL99 (07)

FIG. 5: Prediction for the azimuthal moment $M_\gamma^\perp$ at $\sqrt{s} = 200$ GeV, as a function of $\eta_p$, integrated over $-1 \leq \eta_j \leq 0$ and $0.02 \leq x_j \leq 0.05$. Solid line: using gluonic-pole cross sections. Dashed line: using standard partonic cross sections. Dotted line: maximum contribution from the gluon Sivers function (absolute value). Dot-dashed line: maximum contribution from the Boer-Mulders function (absolute value).
Example of phenomenology

A.B., D’Alesio, Bomhof, Mulders, Murgia, PRL99 (07)

“Standard” universality

FIG. 5: Prediction for the azimuthal moment $M^\gamma_N$ at $\sqrt{s} = 200$ GeV, as a function of $\eta_j$, integrated over $-1 \leq \eta_j \leq 0$ and $0.02 \leq x_\perp \leq 0.05$. Solid line: using gluonic-pole cross sections. Dashed line: using standard partonic cross sections. Dotted line: maximum contribution from the gluon Sivers function (absolute value). Dot-dashed line: maximum contribution from the Boer-Mulders function (absolute value).
A. B., D’Alesio, Bomhof, Mulders, Murgia, PRL 99 (07)

“Standard” universality

“Generalized” universality
Conclusions
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- At the present state of the art, we are forced to conclude that unintegrated parton distribution functions are NOT universal, since they have different gauge links.
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- No problem has been found with integrated parton distribution functions.
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- No problem has been found with integrated parton distribution functions.
- $k_T$ factorization can still hold in principle, even if the functions are non-universal.
Conclusions

- At the present state of the art, we are forced to conclude that unintegrated parton distribution functions are NOT universal, since they have different gauge links.
- No problem has been found with integrated parton distribution functions.
- $k_T$ factorization can still hold in principle, even if the functions are non-universal.
- The non-universality occurring in some weighted asymmetries can be calculated (and checked).
Outlook
Outlook

- For some reason it could turn out that things “conspire” in such a way that PDFs in different processes can be easily related to each other.
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- Maybe the factorization-breaking effects are negligible
Outlook

- For some reason it could turn out that things “conspire” in such a way that PDFs in different processes can be easily related to each other.
- Maybe the factorization-breaking effects are negligible.
- Pessimistic: in hadrons to hadrons processes many different PDFs are involved and no easy relation between them exists. $k_T$ factorization becomes almost useless in these processes.