Helicity Amplitude Ratios in Exclusive Electroproduction of the $\rho^0$ Meson at HERMES

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Amplitudes and Spin Density Matrices.

What can we learn from helicity amplitude?

HERMES Experiment and data processing.

Kinematic dependences of ratios of helicity amplitudes.

Comparison with H1 results.

Summary.
Amplitudes and Spin Density Matrices in reaction $e + N \rightarrow e' + \rho^0 + N$

$e \rightarrow e' + \gamma^*$ (QED). Spin-density matrix
\[ \rho_{\lambda\gamma',\lambda\gamma}^{U+L}(\epsilon, \Phi) = \rho_{\lambda\gamma',\lambda\gamma}^U + P_{beam}\rho_{\lambda\gamma',\lambda\gamma}^L \]
of the virtual photon is known. U - unpolarized, L - polarized beam

$\gamma^* + N \rightarrow \rho^0 + N \rightarrow \pi^+ + \pi^- + N$ (QCD).
Vector-meson spin-density matrix $\rho_{\lambda V',\lambda V}$ is expressed by helicity amplitudes $F_{\lambda V',\lambda V;\lambda\gamma,\lambda\gamma}(W, Q^2, t')$. In CM frame of $\gamma^* N$ is given by the von Neumann formula:
\[ \rho_{\lambda V',\lambda V} = \frac{1}{2N} \sum_{\lambda\gamma,\lambda\gamma'} \rho_{\lambda\gamma',\lambda\gamma} F_{\lambda V',\lambda V;\lambda\gamma,\lambda\gamma}^L \rho_{\lambda V',\lambda V;\lambda\gamma,\lambda\gamma}^U \]

After decomposition of $\rho_{\lambda\gamma,\lambda\gamma'}^{L+U}$ into the set of nine hermitian matrices $(3 \times 3)\Sigma^\alpha$ ($\alpha = 0 \div 3$ - transv., $4$ - long. $5 \div 8$ - interf.) , when we can not separate transverse and longitudinal photons, Spin Density Matrix Elements (SDMEs) are defined:
\[ r_{\lambda V',\lambda V}^{04} = (\rho_{\lambda V',\lambda V}^0 + \epsilon R \rho_{\lambda V',\lambda V}^4) / (1 + \epsilon R), \]
\[ r_{\lambda V',\lambda V}^\alpha = \begin{cases} \rho_{\lambda V',\lambda V}^\alpha, & \alpha = 1, 2, 3, \\ \frac{\rho_{\lambda V',\lambda V}^\alpha}{\sqrt{1 + \epsilon R}}, & \alpha = 5, 6, 7, 8. \end{cases} \]

$R = \sigma_L / \sigma_T$
Angular distribution in reaction

\[ e + N \rightarrow e' + \rho^0 + N \rightarrow e' + \pi^+ + \pi^- + N \]

\[ \rho^0 \Rightarrow \pi^+ \pi^- \text{ (conservation of } \vec{J} \text{)} \]

\[ |\rho^0; 1m > \rightarrow |\pi^+ \pi^-; 1m > \Rightarrow Y_{1m}(\cos \theta, \phi), \]

\( (m = \pm 1, 0). \) Angular distribution \( \mathcal{W}(\Phi, \phi, \cos \Theta) \)

depends linearly on \( r^\alpha_{\lambda V} \lambda'_{\lambda V} \) and beam polarization \( P_b. \)

For longitudinally polarized beam and unpolarized target there are 23 SDMEs, which are determined from the fit of angular distribution of pions from decay \( \rho^0 \Rightarrow \pi^+ \pi^- \)

In turn, all SDMEs are bilinear combination of helicity amplitudes and in "principle" can also be determined from the fit of angular distribution.
General properties of helicity amplitudes

Total number of amplitudes:
- Initial state 3 spin states of \( \gamma^* \lambda_\gamma = (1, 0, -1) \) and 2 nucleon helicities \( \lambda_N = (\frac{1}{2}, -\frac{1}{2}) \)
- Final state 3 spin states of \( \rho^0 \lambda'_V = (1, 0, -1) \) and nucleons \( \lambda'_N = (\frac{1}{2}, -\frac{1}{2}) \) → 36 amplitudes
- Due to parity conservation:
  \[ F_{-\lambda_V - \lambda'_N; -\lambda_\gamma - \lambda_N} = (-1)(\lambda_V - \lambda'_N) - (\lambda_\gamma - \lambda_N) F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N}, \] → 18 amplitudes.

Helicity amplitude can be decomposed into a sum of an amplitude \( T \) for natural-parity exchange (NPE)(\( P = (-1)^J \)) and an amplitude \( U \) for unnatural-parity exchange (UPE)(\( P = -(-1)^J \)).
\[ F_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} = T_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} + U_{\lambda_V \lambda'_N; \lambda_\gamma \lambda_N} \]

The amplitudes obey the following symmetry relation:
\[ T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} = (-1)^{-\lambda_V + \lambda_\gamma} T_{-\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} = (-1)^{-\lambda'_N + \lambda_N} T_{\lambda_V - \lambda'_N - \lambda_\gamma - \lambda_N} \]
\[ U_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} = -(-1)^{-\lambda_V + \lambda_\gamma} U_{-\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} = -(-1)^{-\lambda'_N + \lambda_N} U_{\lambda_V - \lambda'_N - \lambda_\gamma - \lambda_N} \]

Due to these symmetry relations production of vector meson is described by 10 NPE and 8 UPE amplitudes.
No UPE amplitude exists for the transition \( \gamma_L \rightarrow \rho^0_L \). \( T_{00} \equiv F_{00} \equiv F_{0\frac{1}{2}0\frac{1}{2}} \).
For unpolarized target there is no interference between NPE and UPE amplitudes.

The SDMEs are expressed by 18 complex helicity amplitudes, 36 parameters (real and imaginary parts).
On unpolarized target there is no linear contribution of nucleon-helicity-flip amplitudes to SDMEs (suppressed by factor $(\alpha)^2 = (\frac{\sqrt{-t'}}{M})^2$ \(t' = t - t_{min}\)). This reduces the number of NPE amplitudes to five: Helicity conserving $T_{00}, T_{11}$, helicity non conserving $T_{01}, T_{10}, T_{1-1}$, where we used shorthand notation $T_{\lambda V \lambda \gamma} = T_{\lambda V \frac{1}{2} \lambda \gamma \frac{1}{2}}$. The dominance of diagonal transitions is called s-channel helicity conservation (SCHC).

From the SDME analysis it has been found that for UPE transitions amplitudes obey the following hierarchy: $|U_{01}|^2, |U_{10}|^2, |U_{1-1}|^2 \ll |U_{11}|^2$, we keep only $|U_{11}| = \sqrt{|U_{\frac{1}{2}1\frac{1}{2}}|^2 + |U_{\frac{1}{2}-1\frac{1}{2}}|^2}$. For UPE amplitudes it is not possible to prove the dominance of those without spin flip over those with spin flip.

The hierarchy of amplitudes in the kinematic region of HERMES is:
$|T_{00}|^2 \sim |T_{11}|^2 \gg |U_{11}|^2 > |T_{01}|^2 > |T_{10}|^2 \sim |T_{-1-1}|^2$,

Since SDMEs depend rather on ratios of these complex amplitudes, the number of real parameters which determine all SDMEs is 9 (real and imaginary parts).

Finally, we approximated the SDMEs through 9 real parameters, namely: $Re\{T_{11}/T_{00}\}, Im\{T_{11}/T_{00}\}, Re\{T_{01}/T_{00}\}, Im\{T_{01}/T_{00}\}, Re\{T_{10}/T_{00}\}, Im\{T_{10}/T_{00}\}, Re\{T_{1-1}/T_{00}\}, Im\{T_{1-1}/T_{00}\}, |U_{11}/T_{00}|$ where $|U_{11}/T_{00}|$ is the module of $U_{11}/T_{00}$. 
What can we learn from helicity amplitudes of the process $\gamma^* + N \rightarrow \rho^0 + N$

- NPE ($J^P = 0^+, 1^-, ...$) amplitudes $T_{\lambda_V \lambda_\gamma}$ (Two-gluon exchange = pomeron, $\rho$, $\omega$, $a_2$, ... reggeons = $q\bar{q}$ exchange). UPE ($J^P = 0^-, 1^+, ...$) amplitudes $U_{\lambda_V \lambda_\gamma}$ ($\pi$, $a_1$, $b_1$, ... reggeons = $q\bar{q}$ exchange). In the GPD formalism, NPE amplitudes are described by $H$ and $E$, UPE by $\tilde{H}$, $\tilde{E}$. The amplitude ratios can be used to distinguish between contribution of NPE and UPE processes. For this aim, an amplitude ratio is more convenient than SDMEs as any SDME depends on all amplitude ratios.

- Violation of $s$-channel helicity ($\lambda_V \neq \lambda_\gamma$) can be studied more reliably using amplitude ratios rather than SDMEs. The spin flip amplitudes $T_{01}, T_{10}$ provide information on valence quark motion in vector meson. (They are to be zero in the absence of quark motion in meson). The double spin flip amplitudes $T_{1-1}$ contain information on gluon distribution in nucleon.

- Difference between proton and deuteron results would point to a contribution of $q\bar{q}$-exchange with isospin $I = 1$ and natural parity $P = (-1)^J$ ($\rho$, $a_0$, $a_2$ reggeons).

- The present work is a continuation of the Spin Density Matrix Elements (SDME) analysis published at **EPJ C62(2009) 659**.
Beam $e^\pm$, $P = 27.56$ GeV/c longitudinal polarization $\sim 55\%$.

Target longitudinally, transversely polarized H or D or unpolarized gas target.

Acceptance: $|\Theta_x| < 170$ mrad, $40 < |\Theta_y| < 140$ mrad.

Resolution $\delta P / P \leq 1\%$, $\delta \Theta \leq 0.6$ mrad.

PID: RICH, TRD, Preshower, Calorimeter.
Exclusive $\rho^0$-meson production at HERMES

$W = 3.0 \div 6.5$ GeV, $< W > = 4.9$ GeV total number of events (1996-2005) $W^2 = (q + p)^2$

$Q^2 = 0.5 \div 7.0$ GeV$^2$, $< Q^2 > = 1.95$ GeV$^2$ Deuteron: $\rho^0$ - 16388

$Q^2 = -(k - k')^2$

$x_B = 0.01 \div 0.35$, $< x_B > = 0.08$ Hydrogen: $\rho^0$ - 9860

$x_B = \frac{Q^2}{2pq}$

$0 \leq -t' \leq 0.4$ GeV$^2$, $< -t' > = 0.13$ GeV$^2$ with $t' = t - t_{min}$

$t = (q - v)^2$

$\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$ with $M_X^2 = (p + q - p_{\pi^+} - p_{\pi^-})^2$ and $M_X$ being missing mass, p, q, $p_{\pi^+}$, $p_{\pi^-}$ are 4-momenta of proton, $\gamma^*$ and pions.

$-1.0 < \Delta E < 0.6$ GeV,

$0.6 < M_{\pi\pi} < 1$ GeV,

SIDIS background is subtracted using MC PYTHIA
Amplitude ratios are extracted directly from the measured angular distribution using Binned Maximum Likelihood (BML) method.

3-dimensional matrix \((\cos \Theta, \phi, \Phi)\) of data. \((8 \times 8 \times 8)\) cells.

3-dimensional matrix of fully reconstructed MC events generated with uniform angular distribution

Minimizing the difference between data matrix and MC matrix reweighted by \(\mathcal{W}(\Phi, \phi, \cos \Theta)\) which depends on 5 ratios of helicity amplitudes, i.e. 9 real fitted parameters. Simultaneous fit for data with negative and positive beam helicity (\(< P_b > = 47.0\%)\)

There is agreement of fitted angular distribution with the HERMES data.

The amplitude ratios are extracted with the same BML method as SDMEs EPJ C62(2009) 659.

An example of fitted angular distribution. The blue lines represent isotropic input Monte Carlo distribution as modified by the HERMES acceptance, while the red lines are the results of the fit. \(\Psi = \phi + \Phi\) (SCHC approximation)
No difference between proton and deuteron for amplitude ratio $T_{11}/T_{00}$.

pQCD predicts the following dependence: $T_{11}/T_{00} \propto M_\rho/Q$.

The Q dependence of $T_{11}/T_{00}$ is fitted with $\text{Re}\{T_{11}/T_{00}\} = a/Q$, $\text{Im}\{T_{11}/T_{00}\} = b \cdot Q$. Combined data on proton and deuteron: $a = 1.129 \pm 0.024$ GeV, $\chi^2/N_{df} = 1.02$; $b = 0.344 \pm 0.014$ GeV$^{-1}$, $\chi^2/N_{df} = 0.87$.

Behaviour of $\text{Im}\{T_{11}/T_{00}\}$ is in contradiction with high-Q asymptotics in pQCD.

The $Q^2$ dependence of the phase difference $\delta_{11}$ between the amplitudes $T_{11}$ and $T_{00}$ is given by $\tan \delta_{11} = \text{Im}\{T_{11}/T_{00}\}/\text{Re}\{T_{11}/T_{00}\} = bQ^2/a$.

Phase difference is $\delta_{11} \sim 30^\circ$ at $<Q^2> = 1.95GeV^2$ and grows with $Q^2$ in disagreement with pQCD calculation.
The amplitude $T_{01} = T_{0 1/2}^{1/2}$ describing the transition $\gamma^*_T \rightarrow \rho_0^0$ is the largest SCHC-violating amplitude.

There is no difference between proton and deuteron for amplitude ratio $T_{01}/T_{00}$.

pQCD predicts the following dependence: $\frac{T_{01}}{T_{00}} \propto \sqrt{-t'/Q}$.

The $t'$ dependence of $T_{01}/T_{00}$ is fitted with

$\text{Re}(T_{01}/T_{00}) = a\sqrt{-t'}$, $\text{Im}(T_{01}/T_{00}) = b\sqrt{-t'/Q}$.

Combined proton and deuteron data: $a = 0.399 \pm 0.023 \text{ GeV}^{-1}$, $\chi^2/N_{df} = 0.72$; $b = 0.20 \pm 0.07$, $\chi^2/N_{df} = 1.09$. 

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\(Q^2\) and \(t'\) dependence of \(|U_{11}/T_{00}|\)

**HERMES preliminary**

- **Proton**
- **Deuteron**

**Contrast** with both high-Q asymptotic and one-pion-exchange dominance.

**Unnatural Parity Exchange** is seen here much better than in SDME method.

- No difference between proton and deuteron for amplitude ratio \(|U_{11}/T_{00}|\).
- pQCD predicts the following dependence: \(U_{11}/T_{00} \propto M_\rho/Q\).
- We do not see either \(Q^2\) or \(t'\) dependence: \(|U_{11}|/|T_{00}| = a, a = 0.391 \pm 0.013, \chi^2/N_{df} = 0.44\)
Test of Unnatural-Parity Exchange for $\rho^0$ meson

- Natural and Unnatural Parity Exchanges in the $t$-channel
  NPE: GPD $H, E$ ; $T_{\lambda \rho \lambda \gamma}$
  UPE: GPD $\tilde{H}, \tilde{E}$ ; $U_{\lambda \rho \lambda \gamma}$
  NPE (Pomeron, $\rho, \omega, f_2, a_2, ...$) dominate and
  UPE ($\pi, a_1, b_1...$) are suppressed at high energies

- Signal of UPE in SDME method
  \[
  u_1 = 1 - r_{00}^0 + 2r_{01}^0 - 2r_{11} - 2r_{10},
  \]
  \[
  u_1 = \sum \lambda_N \chi'_N \frac{2\epsilon |U_{10}|^2 + |U_{11} + U_{-11}|^2}{N}
  \]
  where $N = N_T + \epsilon N_L$,
  \[
  N_T = \sum \lambda_N \chi'_N (|T_{11}|^2 + |T_{01}|^2 + |T_{-11}|^2 + |U_{11}|^2 + |U_{01}|^2 + |U_{-11}|^2),
  \]
  \[
  N_L = \sum \lambda_N \chi'_N (|T_{00}|^2 + |T_{01}|^2 + |T_{-10}|^2 + |U_{10}|^2 + |U_{-10}|^2).
  \]

$u_1 = 0.125 \pm 0.021_{\text{stat}} \pm 0.050_{\text{syst}}$ (H),
$u_1 = 0.091 \pm 0.016_{\text{stat}} \pm 0.046_{\text{syst}}$ (D),
$u_1 = 0.106 \pm 0.036_{\text{tot}}$ (H+D)


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Comparison of Hermes and H1 results

H1: Unpolarized beam and unpolarized target (15 SDMEs), $<Q^2> = 3.3$ GeV$^2$.

Assumption: only NPE, all amplitudes are purely imaginary, all amplitude ratios are real.

HERMES: Longitudinally polarized beam and unpolarized target (23 SDMEs).

Both real and imaginary parts of ratios of helicity amplitudes are extracted.

Excellent agreement of amplitude ratios extracted by H1 and HERMES.
Study of electroproduction of $\rho^0$ vector meson on proton and deutron enables to obtain ratios of helicty amplitudes, and to investigate their kinematic dependences.

The kinematic dependences of $\text{Im}\{T_{11}/T_{00}\}$, $|U_{11}/T_{00}|$ are in contradiction with high-Q asymptotics behavior predicted in pQCD. The dependences of $\text{Re}\{T_{11}/T_{00}\}$ and $\text{Im}\{T_{01}/T_{00}\}$ are in agreement with pQCD prediction.

The amplitude ratios for deuterons are compatible with those for protons.

The UPE signal is seen here with very high significance for both proton and deuteron data and with higher precision than that obtained in SDME method.

Violation of S-channel helicity conservation is determined with higher accuracy from studying amplitudes than from SMDEs.