Spin Density Matrix Elements (SDMEs) and Helicity Amplitude Ratios in Exclusive $\rho^0$ Electroproduction at HERMES

- Physics motivation
- $\rho^0$ SDMEs on unpolarized $p$ and $d$ data
  - Validity of $S$-channel helicity conservation approximation
  - Longitudinal-to-transverse cross-section ratios
  - Unnatural parity exchange
- Direct extraction of helicity amplitude ratios on unpolarized $p$ and $d$ data
- SDMEs and asymmetries on transversely polarized $p$ data
- Summary

Alexander Borissov, DESY, on behalf of the HERMES Collaboration
1. \(e \rightarrow e + \gamma^*\) (QED)

Spin-density matrix of the virtual photon is known:

\[
\varrho_{\lambda\gamma\lambda'\gamma'}^{U+L} = \varrho_{\lambda\gamma\lambda'\gamma'}^U + P_{\text{beam}} \varrho_{\lambda\gamma\lambda'\gamma'}^L,
\]

where

\[
\varrho_{\lambda\gamma\lambda'\gamma'}^U(e, \Phi) = \frac{1}{2} \begin{pmatrix}
\frac{1}{\sqrt{\epsilon(1 + \epsilon)}} e^{i\Phi} & \sqrt{\epsilon(1 + \epsilon)} e^{-i\Phi} & -\epsilon e^{-2i\Phi} & 0 \\
-\epsilon e^{2i\Phi} & 2\epsilon & -\sqrt{\epsilon(1 + \epsilon)} e^{-i\Phi} & 0 \\
-\epsilon e^{2i\Phi} & 2\epsilon & -\sqrt{\epsilon(1 + \epsilon)} e^{-i\Phi} & 0 \\
\end{pmatrix},
\]

and

\[
\varrho_{\lambda\gamma\lambda'\gamma'}^L(e, \Phi) = \frac{1}{2} \begin{pmatrix}
\sqrt{1 + \epsilon} & \sqrt{\epsilon e^{-i\Phi}} & 0 & 0 \\
\sqrt{\epsilon e^{i\Phi}} & \sqrt{1 + \epsilon} & 0 & 0 \\
0 & 0 & \sqrt{1 + \epsilon} & \sqrt{\epsilon e^{-i\Phi}} \\
0 & 0 & \sqrt{\epsilon e^{i\Phi}} & \sqrt{1 + \epsilon} \\
\end{pmatrix}.
\]

2. \(\gamma^* + N \rightarrow \rho^0 + N\) (QCD)

The spin density matrix \(\rho_{\lambda V\lambda' V'}\) of \(\rho^0\) meson is related to \(\varrho_{\lambda\gamma\lambda'\gamma'}^{U+L}\), through the von Neumann formula:

\[
\rho_{\lambda V\lambda' V'} = \frac{1}{2N} \sum_{\lambda\gamma\lambda' N;\lambda\gamma\lambda N} F_{\lambda V\lambda' N;\lambda\gamma\lambda N} \varrho_{\lambda\gamma\lambda'\gamma'}^{U+L} \rho_{\lambda' V' N;\lambda\gamma\lambda N}^*.
\]

where \(F_{\lambda V\lambda' N;\lambda\gamma\lambda N}(W, Q^2, t') \equiv F_{\lambda V\lambda\gamma}\) are the helicity amplitude of the \(\gamma^* N \rightarrow \rho^0 N\).

After decomposition of \(\varrho_{\lambda\gamma\lambda'\gamma'}^{U+L}\) into the standard set of nine hermitian matrices \(\Sigma^\alpha (\alpha = 0, 1, \ldots, 8)\), without separation of transverse and longitudinal photons:

\[
r^0_{\lambda V\lambda' V'} = (\rho^0_{\lambda V\lambda' V'} + \epsilon R \rho^4_{\lambda V\lambda' V'})/(1 + \epsilon R),
\]

\[
r^\alpha_{\lambda V\lambda' V'} = \begin{cases} 
\rho^\alpha_{\lambda V\lambda' V'}/(1 + \epsilon R), & \alpha = 1, 2, 3, \\
\sqrt{\epsilon} \rho^\alpha_{\lambda V\lambda' V'}/(1 + \epsilon R), & \alpha = 5, 6, 7, 8.
\end{cases}
\]

On unpolarized target SDMEs are presented according K.Schilling and G.Wolf (Nucl. Phys. B61 (1973) 381), \(\alpha = 04, 1 \div 3, 5 \div 8\) long. or trans. photon, \(\lambda_\rho = -1, 0, 1\) - polarization of \(\rho^0\).

3. \(\rho^0 \Rightarrow \pi^+\pi^-\) (conservation of \(\vec{J}\)), \(|\rho^0; 1m \rightarrow |\pi^+\pi^-; 1m \Rightarrow Y_{1m}(\theta, \phi)|\)
General Properties of Amplitudes

- **Total number of amplitudes.**
  - Initial: 3 spin states of $\gamma^* : (\lambda_\gamma \equiv j = 1, 0, -1)$ and 2 nucleon helicities ($\lambda_N \equiv n = \frac{1}{2}, -\frac{1}{2}$).
  - Final: the same for $\rho^0$.
  - Parity conservation: $T_{im;jn} = T_{-i-m;-j-n} \cdot (-1)^{i-j+m-n}$ → 18 independent amplitudes $T_{im;jn}$, 36 parameters to fit

- **Natural-parity plus Unnatural-parity representation:**
  - $F_{\lambda_\rho \lambda_\gamma} = T_{\lambda_\rho \lambda_\gamma} + U_{\lambda_\rho \lambda_\gamma}$
  - NPE: $T_{im;jn} = \frac{1}{2} [F_{im;jn} + (-1)^{i-j} \cdot F_{-im;-jn}]$, GPDs: no target spin-flip $H$, spin-flip $E$
  - UPE: $U_{im;jn} = \frac{1}{2} [F_{im;jn} - (-1)^{i-j} \cdot F_{-im;-jn}]$, GPDs: no target spin-flip $\tilde{H}$, spin-flip $\tilde{E}$
  - No interference between NPE and UPE contributions to SDMEs for unpolarized target (Schilling, Wolf, 1978) → 10 NPE plus 8 UPE amplitudes

- **On unpolarized target** contribution of NPE amplitudes with nucleon spin flip ($T_{\lambda_\rho -\frac{1}{2}; \lambda_\gamma \frac{1}{2}}$ and $T_{\lambda_\rho \frac{1}{2}; \lambda_\gamma -\frac{1}{2}}$) are to be neglected since no linear contributions to SDMEs.

  $\implies$ Results to 5 NPE and 4 UPE amplitudes:
  - Five NPE amplitudes are $T_{00}, T_{11} = T_{-1-1}, T_{01} = -T_{0-1}, T_{10} = -T_{-10}, T_{1-1} = T_{-11}$
  - Four UPE amplitudes are $U_{11} = -U_{-1-1}, U_{01} = U_{0-1}, U_{10} = U_{-10}, U_{1-1} = -U_{-11}$, as $U_{00} \equiv 0$. 
- SDMEs are measured experimentally at $2 < W < 100$ GeV (CLAS, HERMES, COMPASS, H1, ZEUS)
- calculated in GPD models from the helicity amplitudes, c.f. GK model for HERMES kinematics

- Number of SDMEs depends on the beam and target polarization (M. Diehl, JHEP09 (2007) 064, arXiv:0704.1565v2)

<table>
<thead>
<tr>
<th>Beam, Target</th>
<th>$UU$</th>
<th>$UL$</th>
<th>$UT$</th>
<th>$LU$</th>
<th>$LL$</th>
<th>$LT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of SDMEs</td>
<td>15</td>
<td>14</td>
<td>30</td>
<td>8</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

$U$ is denoted as Unpolarized, $L$ as a Longitudinal and $T$ as a Transverse polarization.

- Total number of fit parameters for amplitude ratios $\text{Re(Im)}(F_{ij}/T_{00})$ is 34. But due to the hierarchy, nine real parameters are extracted on unpolarized target instead of 23 $UU + LU$ SDMEs.

- In case of discrepancy between theoretical calculation and experimentally obtained SDME, it is not clear which particular amplitudes are not well predicted by a theoretical model.

- The SDMEs can be calculated from the extracted helicity amplitude ratios. The region of the directly extracted SDME values is greater than the region for SDME values calculated in the amplitudes.

$\Rightarrow$ The amplitude ratios are more advanced and representative than SDMEs!
Transverse Target Polarization Asymmetry $A_{UT}$ and/or UT-SDMEs

Goeke, Polyakov, Vanderhaeghen (2001): $A_{UT}^{0} \propto |S_{T}| \sin(\phi - \phi_{s})EH$

$$A_{UT}(\phi, \phi_{s}) = \frac{\sigma_{UT}}{\sigma_{UU}} = \frac{[\sigma(\phi, \phi_{s}) - \sigma(\phi, \phi_{s} + \pi)]/P_{T}}{\int[\sigma(\phi, \phi_{s}) + \sigma(\phi, \phi_{s} + \pi)]d\phi/(2\pi)} \approx \frac{\sqrt{t_{0}-t}}{m_{p}} \frac{\text{Im}(E_{V}^{*}H_{V})}{|H_{V}|^{2}} = \frac{\sqrt{t_{0}-t}}{m_{p}} |\frac{E_{V}}{H_{V}}| \sin \delta_{V},$$

where $\delta_{V} = \arg(H_{V}/E_{V})$, is relative phase between $H_{V}$ and $E_{V}$

\[\Rightarrow\] Access to GPD function $E$ which is sensitive to the angular momentum of quarks and gluons

Ji’s sum rule: $J^{a} = \frac{1}{2} \int dxx[H^{a}(x, 0, 0) + E^{a}(x, 0, 0)]$, where $a = u, d, s$ quarks


$\sin(m_{s} \cdot \phi + n_{s} \cdot \phi_{s})$ moments, see talk of I. Hristova.


GPD based calculations of $A_{UT}^{V}$:


- D. Ivanov (arXiv:0712.3193, and HERMES seminar, DESY, 9.12.2008): stable results for a resummation of NLO amplitudes for vector mesons are presented for fixed target experiments only.

- S. V. Goloskokov, P. Kroll, (Eur. Phys. J. C 53(2008) 367; and arXiv:0809.4126) LO, at $W = 5$ GeV, $Q^{2} \geq 3$ GeV$^{2}$, $0 < -t < 0.4$ GeV$^{2}$, i.e. at HERMES kinematic conditions, $V = \rho^{0}, \phi, \omega, \rho^{+}, K^{*0}$

\[\Rightarrow\] Comparison of HERMES data with the GPD based calculations
HERMES Detector was Two Identical Halves of Forward Spectrometer

- $e^\pm$ beam, $P = 27.6$ GeV/c, longitudinal polarization $\sim 55\%$
- $\sim 80\%$ longitudinally, transversely polarized hydrogen, or unpolarized hydrogen or deuterium targets
- Acceptance: $40 < \Theta < 220$ mrad, $|\Theta_x| < 170$ mrad, $40 < |\Theta_y| < 140$ mrad
- Resolution: $\delta p/p \leq 1\%, \delta \Theta \leq 0.6$ mrad
Data on Exclusive $\rho^0$ Meson Production $e + p \rightarrow e' + p' + \rho^0 \rightarrow \pi^+\pi^-$

Clean exclusive peak of missing energy $\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$, background is subtracted using PYTHIA

![Graph of $\Delta E$ vs. Entries](image)

![Graph of $M_{\pi\pi}$ vs. Events](image)

**Kinematics:**
- $Q^2 = 0.5 \div 7.0 \text{ GeV}^2, \langle Q^2 \rangle = 2.3 \text{ GeV}^2$
- $W = 3.0 \div 6.5 \text{ GeV}, \langle W \rangle = 4.9 \text{ GeV}$
- $x_{Bj} = 0.01 \div 0.35, \langle x_{Bj} \rangle = 0.07$
- $-t' = 0 \div 0.4 \text{ GeV}^2, \langle -t' \rangle = 0.13 \text{ GeV}^2$
Fit of Angular Distributions Using Max. Likelihood Method in MINUIT

Fit of 23 SDMEs after full detector simulation done using initial uniform angular distribution

- Binned Maximum Likelihood Method: $8 \times 8 \times 8$ bins of $\cos(\Theta)$, $\phi$, $\Phi$.
- Simultaneous fit of 23 SDMEs: $r_{ij}^\alpha = W(\Phi, \phi, \cos \Theta)$ for data with negative and positive beam helicity ($\langle |P_b| \rangle = 53.5\%$, $\Psi = \Phi - \phi$).
- 15 “unpolarized” plus, for the first time, 8 “polarized” SDMEs.

$\Rightarrow$ Full agreement of the fitted angular distributions with data
Function for the Fit of 23 SDME $r_{i,j}^{\alpha}$

\[
W(\cos \Theta, \phi, \Phi) = W^{\text{unpol}} + W^{\text{long.pol}},
\]

\[
W^{\text{unpol}}(\cos \Theta, \phi, \Phi) = \frac{3}{8\pi^2} \left[ \frac{1}{2}(1 - r_{00}^{04}) + \frac{1}{2}(3r_{00}^{04} - 1) \cos^2 \Theta - \sqrt{2}\text{Re}\{r_{10}^{04}\} \sin 2\Theta \cos \phi - r_{1-1}^{04}\sin^2 \Theta \cos 2\phi - \epsilon \cos 2\Phi \left( r_{11}^{11} \sin^2 \Theta + r_{00}^{10} \cos^2 \Theta - \sqrt{2}\text{Re}\{r_{10}^{11}\} \sin 2\Theta \cos \phi - r_{1-1}^{11}\sin^2 \Theta \cos 2\phi \right) \right.
\]

- \epsilon \sin 2\Phi \left( \sqrt{2}\text{Im}\{r_{10}^{10}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{2}\} \sin^2 \Theta \sin 2\phi \right)

+ \sqrt{2}\epsilon(1 + \epsilon) \cos \Phi \left( r_{11}^{5} \sin^2 \Theta + r_{00}^{5} \cos^2 \Theta - \sqrt{2}\text{Re}\{r_{10}^{5}\} \sin 2\Theta \cos \phi - r_{1-1}^{5}\sin^2 \Theta \cos 2\phi \right)

+ \sqrt{2}\epsilon(1 + \epsilon) \sin \Phi \left( \sqrt{2}\text{Im}\{r_{10}^{7}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{6}\} \sin^2 \Theta \sin 2\phi \right)
\]

\[
W^{\text{long.pol}}(\cos \Theta, \phi, \Phi) = \frac{3}{8\pi^2} P_{\text{beam}} \left[ \sqrt{1 - \epsilon^2} \left( \sqrt{2}\text{Im}\{r_{10}^{3}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{3}\} \sin^2 \Theta \sin 2\phi \right) \right.
\]

+ \sqrt{2}\epsilon(1 - \epsilon) \cos \Phi \left( \sqrt{2}\text{Im}\{r_{10}^{7}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{7}\} \sin^2 \Theta \sin 2\phi \right)

+ \sqrt{2}\epsilon(1 - \epsilon) \sin \Phi \left( r_{11}^{8} \sin^2 \Theta + r_{00}^{8} \cos^2 \Theta - \sqrt{2}\text{Re}\{r_{10}^{8}\} \sin 2\Theta \cos \phi - r_{1-1}^{8}\sin^2 \Theta \cos 2\phi \right)
\]

\[\Rightarrow \quad \text{"Polarized" SDMEs are measurable with longitudinally polarized beam and } \epsilon < 1\]
\( \rho^0 \) SDMEs According to Hierarchy of Amplitudes with(out) Helicity Flip

- A, \( \gamma^*_L \to \rho^0_L \) and \( \gamma^*_T \to \rho^0_T \)
  \[ |T_{11}|^2 \propto 1 - r_{00}^{04} \propto r_{1-1}^1 \propto -\text{Im}\{r_{1-1}^2\} \]

- B, Interference: \( \gamma^*_L, \rho^0_T \)
  \[ \Re\{T_{00}T_{11}^*\} \propto \Re\{r_{10}^5\} \propto -\text{Im}\{r_{10}^6\} \]
  \[ \Im\{T_{11}T_{00}^*\} \propto \Im\{r_{10}^8\} \propto \Re\{r_{10}^8\} \]

- C, Spin Flip: \( \gamma^*_T \to \rho^0_L \)
  \[ \Re\{T_{11}T_{01}^*\} \propto \Re\{r_{10}^5\} \propto \Re\{r_{10}^8\} \]
  \[ \Re\{T_{01}T_{00}^*\} \propto r_{00}^5 \]
  \[ |T_{01}|^2 \propto r_{00}^5 \]
  \[ \Im\{T_{01}T_{11}^*\} \propto \Im\{r_{10}^3\} \]
  \[ \Im\{T_{01}T_{00}^*\} \propto r_{00}^8 \]

- D, Spin Flip: \( \gamma^*_L \to \rho^0_T \)
  \[ \Re\{T_{10}T_{11}^*\} \propto r_{11}^5 \propto r_{1-1}^1 \propto \Im\{r_{1-1}^6\} \]
  \[ \Im\{T_{10}T_{11}^*\} \propto \Im\{r_{1-1}^7\} \propto r_{11}^8 \propto r_{1-1}^8 \]

- E, Spin Flip: \( \gamma^*_T \to \rho^0_T \)
  \[ \Re\{T_{1-1}T_{11}^*\} \propto r_{1-1}^{04} \propto r_{11}^1 \]
  \[ \Im\{T_{1-1}T_{11}^*\} \propto \Im\{r_{1-1}^3\} \]

\[ \Rightarrow \text{ Hierarchy of } \rho^0 \text{ amplitudes: } |T_{00}| \sim |T_{11}| \gg |T_{01}| > |T_{10}| \gtrsim |T_{1-1}|, \ (0 \to L, 1 \to T) \]

(HERMES collab. EPJ C (in press); arXiv:0901.0701)
Observation of Unnatural Parity Exchange (UPE) in $\rho^0$ Leptoproduction

- Unnatural parity exchange is mediated by pseudoscalar or axial meson: $J^P = 0^-, 1^+$, e.g. $\pi, a_1, b_1 \rightarrow$ only quark-exchange contribution

- No interference between NPE and UPE contributions on unpolarized target

- Extracted from SDMEs:

  \[ u_1 = 1 - r_{00}^{04} + 2r_{1-1}^{04} - 2r_{11}^{1} - 2r_{1-1}^{1} \]

  \[ u_1 \propto \epsilon |U_{10}|^2 + 2|U_{11} + U_{1-1}|^2 \approx 2|U_{11}|^2 \]

  p: \[ u_1 = 0.125 \pm 0.021_{stat} \pm 0.050_{syst} \]

  d: \[ u_1 = 0.091 \pm 0.016_{stat} \pm 0.046_{syst} \]

  p+d: \[ u_1 = 0.106 \pm 0.036 \]

  \[ \Rightarrow \] No $Q^2$ and $t'$ dependences observed for $u_1$
World data on Unnatural Parity Exchange and Spin-Flip Contributions for $\rho^0$

- $\langle u_{1}^{\text{low}-W} \rangle = 0.70 \pm 0.16$

$\Rightarrow$ **W-dependence of** $u_1$

- $u_2 + iu_3 \propto (U_{11} + U_{1-1}) \ast U_{10}$

$\Rightarrow$ **W-dependence of** $u_1$

- $u_2 = r_{11}^5 + r_{1-1}^5$
  - **p:** $u_2 \approx -0.011 \pm 0.013$
  - **d:** $u_2 \approx -0.008 \pm 0.011$

- $u_3 = r_{11}^8 + r_{1-1}^8$
  - **p:** $u_3 \approx 0.055 \pm 0.050$
  - **d:** $u_3 \approx -0.040 \pm 0.040$

$\Rightarrow$ **Indication on hierarchy of** $\rho^0$ **UPE amplitudes:**

$|U_{11}| \gg |U_{10}| \sim |U_{01}|$
Presented commonly measured $R^{04} = \frac{1}{\epsilon} \frac{r_{00}^{04}}{1-r_{00}^{04}}$,

$r_{00}^{04} = \sum \{ \epsilon |T_{00}|^2 + |T_{01}|^2 + |U_{01}|^2 \} / \sigma_{tot}$

$\sigma_{tot} = \epsilon \sigma_L + \sigma_T$

$\sigma_T = \sum \{|T_{11}|^2 + |T_{01}|^2 + |T_{1-1}|^2 + |U_{11}|^2 \}$

$\sigma_L = \sum \{|T_{00}|^2 + 2|T_{10}|^2 \}$

Due to the helicity-flip and unnatural parity amplitudes $R^{04}$ depends on kinematic conditions, and is not identical to $R \equiv |T_{00}|^2 / |T_{11}|^2$ for SCHC and NPE dominance.

$\Rightarrow$ HERMES $\rho^0$ data on $R^{04}$ indicate $R(W)$-dependence

(HERMES collab. EPJ C (in press); arXiv:0901.0701)
$R_0 \equiv |T_{00}|^2 / |T_{11}|^2$

without NPE contribution:

$R_{NPE} \approx R^{04}[1 + 0.5u_1(1 + \epsilon R^{04})]$

Fit of HERMES and ZEUS data:

$R(Q^2) = c_0\left(\frac{Q^2}{M_V^2}\right)^{c_1}$

HERMES:

$c_0 = 0.56 \pm 0.08, \quad c_1 = 0.47 \pm 0.12, \quad \chi^2/d.o.f. = 0.45$

ZEUS:

$c_0 = 0.69 \pm 0.22, \quad c_1 = 0.59 \pm 0.15, \quad \chi^2/d.o.f. = 0.15$

$\Longrightarrow W$-dependence of $c_0$ and $c_1$
$\rho^0$ SDMEs Compared to GK Model Calculations


$1 - r_{00}^{04} \propto r_{1-1}^1 \propto -Im\{r_{1-1}^2\} \propto |T_{11}|^2$

i.e. amplitudes for $\gamma^*_L \rightarrow \rho^0_L$, $\gamma^*_T \rightarrow \rho^0_T$

- W=90 GeV
- W=10 GeV, diamond: COMPASS
- W=5 GeV, circle: HERMES PRELIMINARY

$\Rightarrow$ Fair agreement with data

Disagreement of calculations for SDMEs

$Re \ r_{10}^5$ and $Im \ r_{10}^6$ corresponding to interference of $\gamma^*_L$, $\rho^0_T$ amplitudes, i.e. phase difference between $T_{11}$ and $T_{00}$
Phase Difference $\delta$ between $T_{11}$ and $T_{00}$ amplitudes

\[
\sin \delta = \frac{2\sqrt{\epsilon (\text{Re}\{r_{10}^8\} + \text{Im}\{r_{10}^7\})}}{\sqrt{r_{00}^{04}(1 - r_{00}^{04} + r_{1-1}^1 - \text{Im}\{r_{1-1}^2\})}}.
\]

$\rho^0$ p: $\delta = 30.6 \pm 5.0_{\text{stat}} \pm 2.4_{\text{syst}}$ deg

$\rho^0$ d: $\delta = 36.3 \pm 3.9_{\text{stat}} \pm 1.7_{\text{syst}}$ deg

But in GK model $\delta = 3.1$ deg at $W=5$ GeV

$\implies$ Indication on $Q^2$ dependence of $\delta$

(HERMES collab. EPJ C (in press); arXiv:0901.0701)
Equations “SDMEs-from-Amplitudes”

\( \mathbf{A} : \gamma^*_T \to \rho^T_0 \) and \( \gamma^*_L \to \rho^L_0 \)

\[
\begin{align*}
\rho_{00}^{04} &= \sum \{ \varepsilon |T_{00}|^2 + |T_{01}|^2 + |U_{01}|^2 \} / N_{\text{full}}, \\
r_{11}^{i-1} &= \frac{1}{2} \sum \{ |T_{11}|^2 + |T_{1-1}|^2 - |U_{11}|^2 - |U_{1-1}|^2 \} / N_{\text{full}}, \\
\text{Im}\{ r_{1-1}^2 \} &= \frac{1}{2} \sum \{ -|T_{11}|^2 + |T_{1-1}|^2 + |U_{11}|^2 - |U_{1-1}|^2 \} / N_{\text{full}}, \\
\end{align*}
\]

\( \mathbf{B} : \) interference of \( \gamma^*_L \to \rho^L_0 \) and \( \gamma^*_T \to \rho^T_0 \)

\[
\begin{align*}
\text{Re}\{ r_{10}^5 \} &= \frac{1}{\sqrt{8}} \sum \text{Re}\{ 2T_{10}T_{01}^* + (T_{11} - T_{1-1})T_{00}^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{10}^6 \} &= \frac{1}{\sqrt{8}} \sum \text{Re}\{ 2U_{10}U_{01}^* - (T_{11} + T_{1-1})T_{00}^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{10}^7 \} &= \frac{1}{\sqrt{8}} \sum \text{Im}\{ 2U_{10}U_{01}^* + (T_{11} + T_{1-1})T_{00}^* \} / N_{\text{full}}, \\
\text{Re}\{ r_{10}^8 \} &= \frac{1}{\sqrt{8}} \sum \text{Im}\{ -2T_{10}T_{01}^* + (T_{11} - T_{1-1})T_{00}^* \} / N_{\text{full}}, \\
\end{align*}
\]

\( \mathbf{C} : \gamma^*_T \to \rho^T_0 \)

\[
\begin{align*}
\text{Re}\{ r_{10}^5 \} &= \sum \text{Re}\{ \varepsilon T_{10}T_{00}^* + \frac{1}{2} T_{01}(T_{11} - T_{1-1})^* + \frac{1}{2} U_{01}(U_{11} + U_{1-1})^* \} / N_{\text{full}}, \\
\text{Re}\{ r_{10}^6 \} &= \frac{1}{2} \sum \text{Re}\{ -T_{01}(T_{11} - T_{1-1})^* + U_{01}(U_{11} + U_{1-1})^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{10}^7 \} &= \frac{1}{2} \sum \text{Re}\{ T_{01}(T_{11} + T_{1-1})^* - U_{01}(U_{11} + U_{1-1})^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{10}^8 \} &= \sqrt{2} \sum \text{Re}\{ T_{01}T_{00}^* \} / N_{\text{full}}, \\
\end{align*}
\]

\( \mathbf{D} : \gamma^*_L \to \rho^L_0 \)

\[
\begin{align*}
\rho_{00}^{i1} &= \frac{1}{\sqrt{2}} \sum \text{Re}\{ T_{10}(T_{11} - T_{1-1})^* + U_{10}(U_{11} - U_{1-1})^* \} / N_{\text{full}}, \\
r_{1-1}^{5-1} &= \frac{1}{\sqrt{2}} \sum \text{Re}\{ -T_{10}(T_{11} - T_{1-1})^* + U_{10}(U_{11} - U_{1-1})^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{1-1}^6 \} &= \frac{1}{\sqrt{2}} \sum \text{Re}\{ T_{10}(T_{11} + T_{1-1})^* - U_{10}(U_{11} + U_{1-1})^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{1-1}^7 \} &= \frac{1}{\sqrt{2}} \sum \text{Re}\{ T_{10}(T_{11} + T_{1-1})^* - U_{10}(U_{11} + U_{1-1})^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{1-1}^8 \} &= -\frac{1}{\sqrt{2}} \sum \text{Re}\{ T_{10}(T_{11} - T_{1-1})^* + U_{10}(U_{11} - U_{1-1})^* \} / N_{\text{full}}, \\
\end{align*}
\]

\( \mathbf{E} : \gamma^*_T \to \rho^T_0 \)

\[
\begin{align*}
\rho_{00}^{04} &= \sum \text{Re}\{ -\varepsilon |T_{10}|^2 + \varepsilon |U_{10}|^2 + T_{1-1}T_{11}^* - U_{1-1}U_{11}^* \} / N_{\text{full}}, \\
r_{11}^{i1} &= \sum \text{Re}\{ T_{1-1}T_{11}^* + U_{1-1}U_{11}^* \} / N_{\text{full}}, \\
\text{Im}\{ r_{1-1}^3 \} &= \sum \text{Re}\{ T_{1-1}T_{11}^* - U_{1-1}U_{11}^* \} / N_{\text{full}}, \\
\end{align*}
\]

where \( N_{\text{full}} \) is normalized total \( \rho^0 \) production cross section.
Direct Extraction of Amplitude Ratios from Angular Distributions

- The SDMEs are expressed as the ratio of the sums of the bilinear products of helicity amplitudes.

- Dividing both the numerators and denominator of the equations “SDMEs-from-amplitudes” by $|T_{00}|^2$ formulas for the SDMEs expressed through the amplitude ratios.

- From the analysis of SDMEs, UPE SCHnC amplitudes are negligible: $U_{01} \approx U_{10} \approx U_{1-1} \approx 0$.

- Putting these expressions for the SDMEs into formulas “angular distributions-from-SDMEs” we fit the measured angular distribution considering nine amplitude ratios as free parameters:

\[
A_1 = \text{Re}\{T_{11}/T_{00}\}, \quad A_2 = \text{Im}\{T_{11}/T_{00}\}, \\
A_3 = \text{Re}\{T_{01}/T_{00}\}, \quad A_4 = \text{Im}\{T_{01}/T_{00}\}, \\
A_5 = \text{Re}\{T_{10}/T_{00}\}, \quad A_6 = \text{Im}\{T_{10}/T_{00}\}, \\
A_7 = \text{Re}\{T_{1-1}/T_{00}\}, \quad A_8 = \text{Im}\{T_{1-1}/T_{00}\}, \\
A_9 = |U_{11}/T_{00}|,
\]

where $|U_{11}|^2 = |U_{1\frac{1}{2};1\frac{1}{2}}|^2 + |U_{1\frac{1}{2};1-\frac{1}{2}}|^2$, i.e. UPE target-spin-flip amplitudes are not neglected.

- $Q^2$ and $t'$-dependences are studied → 2-dimensional ($Q^2$, $t'$) binning is used. $Q^2$ bins are the following: $0.5 \div 1.0 \div 1.4 \div 2.0 \div 7.0$ GeV^2, $-t'$ bins are: $0.0 \div 0.04 \div 0.10 \div 0.20 \div 0.40$ GeV^2.
\( Q^2 \) dependence of Amplitude Ratios \( \text{Re}(\text{Im})(T_{11}/T_{00}) \)

- pQCD prediction (Ivanov, Kirshner '98) and GPD GK model: \( T_{11}/T_{00} \propto M_V/Q \)

- Fits of \( Q \) dependence: \( \text{Re}\{T_{11}/T_{00}\} = a/Q, \quad \text{Im}\{T_{11}/T_{00}\} = b \cdot Q \)
  
  Combined \( p + d \) data: \( a = 1.129 \pm 0.024 \text{ GeV}, \quad \chi^2/N_{df} = 1.02 \)
  \( b = 0.344 \pm 0.014 \text{ GeV}^{-1}, \quad \chi^2/N_{df} = 0.87 \)

- Fit results are in full agreement with the another representation from the SDME analysis, where the phase difference \( \delta_{11} \sim 30^\circ \) and grows with \( Q^2 \)
$Q^2$ and $t'$ dependence of Amplitude Ratios $|U_{11}/T_{00}|$

- pQCD prediction: $U_{11}/T_{00} \propto M_V/Q$

- Fit: $|U_{11}/T_{00}| = a$, $a = 0.391 \pm 0.013$, $\chi^2/N_{df} = 0.44$

- Contribution of Unnatural Parity Exchange is observed with much better accuracy than in SDME method

$\Rightarrow$ Neither $Q^2$ nor $t'$ dependence visible
$t'$ dependence of Amplitude Ratios $\text{Re(Im)}(T_{01}/T_{00})$

- HERMES preliminary

$e^+p \rightarrow e^+\rho^0p$ ($e^-\bar{p} \rightarrow e^-\rho^0\bar{p}$)

- Fit of $t'$ dependence:
  \[
  \text{Re}\{T_{01}/T_{00}\} = a\sqrt{-t'}, \quad \text{Im}\{T_{01}/T_{00}\} = b\sqrt{-t'}/Q
  \]
  Combined $p + d$ data:
  \[
  a = 0.399 \pm 0.023 \text{ GeV}^{-1}, \quad \chi^2/N_{df} = 0.72;
  \]
  \[
  b = 0.20 \pm 0.07, \quad \chi^2/N_{df} = 1.09.
  \]

$\Rightarrow$ Re and Im parts of $T_{01}/T_{00}$ have different slopes of $t'$.

- Correspondingly, phase difference $\delta_{01}$ decreases with $Q^2$ and $\delta_{01} = (29 \pm 9)^\circ$ at $Q^2 = 0.8$ GeV$^2$.

- pQCD prediction (Ivanov, Kirshner'98) and GPD GK model: $T_{01}/T_{00} \propto \sqrt{-t'}/Q$
New formalism for L&T polarized targets \cite{M.Diehl, JHEP09 (2007) 064, arXiv:0704.1565v2}, interference between NPE and UPE amplitudes accounted, $\phi \pm \phi_s$ distributions are in the fit function.

- Notation: $F_{\gamma^* p}^{\rho_0 p'} \equiv F_{\mu \lambda}^{\nu \sigma}$

$$
\rho_{\mu \mu', \lambda \lambda'}^{\nu \nu'} = (N_T + \epsilon N_L)^{-1} \sum_\sigma F_{\mu \lambda}^{\nu \sigma} (F_{\mu' \lambda'}^{\nu' \sigma'})^*
$$

$$
\begin{align*}
    s_{\mu \mu', \lambda \lambda'}^{\nu \nu'} &= \frac{1}{2} (\rho_{\mu \mu', ++}^{\nu \nu'} + \rho_{\mu \mu', --}^{\nu \nu'}) \\
    n_{\mu \mu', \lambda \lambda'}^{\nu \nu'} &= \frac{1}{2} (\rho_{\mu \mu', +}^{\nu \nu'} - \rho_{\mu \mu', -}^{\nu \nu'})
\end{align*}
$$

- $n$ and $s$ are for normal and sideways target polarization with respect to the direction of the virtual photon and electron scattering plane.

- Sub(super) scripts refer to the helicity of the virtual photon ($\rho^0$ meson) in the amplitudes that occur in the SDME.
$\rho^0$ SDMEs on Transversely Polarized Proton  
(HERMES collab. arXiv:0906.5160 DESY-09-094)

- $P_T = 0.724 \pm 0.059$ with scale uncertainty 8.1%

- Indication on nonzero SDMEs:
  \begin{align*}
  \text{Im}(s^{-+}), \quad \text{Im}(s_{0+}^{0+} - s_{0+}^{-0}) \text{ due to the interference of NPE and UPE amplitudes on the trasversely polarized target,}
  \\
  \text{and Im}(n_{0+}^{00}) \text{ due to violation of SCHC.}
  \end{align*}

- $\implies$ Small values of all 30 SDMEs
$\rho^0$ Transverse Target Polarization Asymmetry

(HERMES collab. arXiv:0906.5160 DESY-09-094)

- Average kinematics:
  \[
  \langle -t' \rangle = 0.13 \text{ GeV}^2 \\
  \langle x_B \rangle = 0.08 \\
  \langle Q^2 \rangle = 1.95 \text{ GeV}^2
  \]

- $\sigma_L$ and $\sigma_T$ separation
  done using the $\rho^0$ UT-SDMEs:

  \[
  A_{UT}^{(\gamma^* L \rightarrow \rho^0 L), \sin(\phi-\phi_s)} = \frac{\text{Im}(n_{00}^{++} + \epsilon_{00}^{00})}{(u_{00}^{00} + \epsilon u_{00}^{00})} \\
  A_{UT}^{(\gamma^* T \rightarrow \rho^0 T), \sin(\phi-\phi_s)} = \frac{\text{Im}(n_{00}^{++} + n_{--}^{++} + 2\epsilon_{00}^{++})}{(1-u_{00}^{00} + \epsilon u_{00}^{00})}
  \]

\[\Rightarrow\] Compatible with zero overall value for leading amplitude: $A_{UT}^{\rho^0} = -0.033 \pm 0.058$
Calculations of $\rho^0$ Transverse Target Polarization Asymmetry


S. V. Goloskokov, P. Kroll arXiv:0809.4126

$A_{UT}$ at $Q^2 = 4 \text{GeV}^2$, $t = -0.4 \text{GeV}^2$

$A_{UT}^\rho$ is also small in calculations

HERMES PRELIMINARY
Asymmetries of $\omega$ Meson Produced on Transversely Polarized Proton

In leading twist:

$$A_{UT}(\phi, \phi_s) = \frac{[\sigma(\phi, \phi_s) - \sigma(\phi, \phi_s + \pi)]/P_T}{\int[\sigma(\phi, \phi_s) + \sigma(\phi, \phi_s + \pi)]d\phi/(2\pi)} = \sum_{m,n} A_{UT}^\sin(m\phi + n\phi_s) \sin(m\phi + n\phi_s)$$

First indication on negative $A_{UT}$ in $\omega$ meson leptoproduction:

$$A^\omega_{UT} \sin(\phi - \phi_s) = -0.22 \pm 0.16_{st} \pm 0.11_{syst}$$

No contradiction with small value of $A^{\rho^0}_{UT}$ due to different contributions of $u$ and $d$ quarks in GPD $E^\rho$:

$$A^\sin(\phi - \phi_s)_{UT}(\rho^0) \propto \sqrt{-t'} \frac{\text{Im}\left\{\frac{2E^u + E^d}{2H^u + H^d}\right\}}{M}$$

$$A^\sin(\phi - \phi_s)_{UT}(\omega) \propto \sqrt{-t'} \frac{\text{Im}\left\{\frac{2E^u - E^d}{2H^u - H^d}\right\}}{M}$$

Note agreement with the calculated for HERMES $A^\omega_{UT} \approx -0.10$

( S. V. Goloskokov, P. Kroll arXiv:0809.4126)

and also predicted $A^{\rho^+}_{UT} \approx 0.40$. 

<table>
<thead>
<tr>
<th>HERMES PRELIMINARY</th>
<th>e+p↑ → e+p+ω</th>
<th>8.2% scale uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{UT}^\sin(\phi - \phi_s)$</td>
<td>$A_{UT}^\sin(\phi + \phi_s)$</td>
<td>$A_{UT}^\sin(\phi_s)$</td>
</tr>
</tbody>
</table>

overall $-t'$ [GeV$^2$] $x_B$ $Q^2$ [GeV$^2$]
Summary

• 15 unpolarized and, for the first time, 8 polarized $\rho^0$ SDMEs are obtained on unpolarized proton and deuteron.
  – No statistically significant difference between proton and deuteron data is observed.
  – Violation of $S$-channel helicity is confirmed in $\rho^0$ electroproduction with high accuracy.
  – Various versions of $R = \sigma_L/\sigma_T$ are determined, indication is found for a $W$ dependence of $R$.
  – Unnatural parity exchange contribution in $\rho^0$ production is seen for the combined data on the proton and deuteron SDMEs with $3 \sigma_{tot}$.

• For the first time, $Q^2$ and $t'$ dependences of the Re&Im parts of the ratios of amplitudes: $T_{11}/T_{00}, T_{01}/T_{00},$ and $|U_{11}/T_{00}|$ are measured for $\rho^0$ electroproduction.
  – $Q^2$ dependence of $\text{Im}(T_{11}/T_{00})$ differs from pQCD prediction and confirm increase of phase difference with $Q^2$, with better precision than at SDMEs approach
  – $t'$ dependence of $\text{Im}(T_{01}/T_{00})$ also indicates different phase of $T_{01}$ amplitude relative $T_{00}$.
  – No $Q^2$ and $t'$ dependences of unnatural parity exchange amplitude in $\rho^0$ meson production is found.

• For the first time, HERMES measured 30 SDMEs for $\rho^0$ and single spin asymmetry for $\rho^0$ and $\omega$ on transversely polarized proton.
  – Small values of SDMEs and $A_{UT}$ are observed for $\rho^0$ electroproduction.
  – Indication on negative $A_{\omega UT}^\omega$ is obtained, which is in agreement with GK model GPD based calculations.

$\implies$ HERMES data provide tests and constrains on GPDs $H$, $\hat{H}$ and $E$. 