Determination of the Structure Function $F_2$ at hermes

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Probing the Structure of Nucleons

- **Deep-inelastic scattering** (DIS) plays major role in understanding of nucleon structure
- Lepton-nucleon scattering **cleanest way** to probe substructure of nucleon
- Exchange of virtual boson, **breakup** and hadronization in DIS regime

For given $s$, two kinematic variables completely describe the scattering process in the inclusive analysis, e.g.:

\[ Q^2 = -q^2 = (k-k')^2 = 4E'E\sin^2 \theta \frac{\theta}{2} \quad \text{photon virtuality} \]
\[ \nu = \frac{P\cdot q}{M} = E - E' \quad \text{photon energy (lab)} \]

Invariant mass of hadronic final state:
\[ W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2 \]

Resolution of deep-inelastic scattering:
\[ \lambda = \frac{1}{|q|} = \frac{1}{\sqrt{\nu^2 + Q^2}} \approx \frac{2Mx}{Q^2} \]
Structure Functions of the Nucleon

Deep-inelastic scattering in the \textit{one-photon} exchange approximation can be written as:

\[
\frac{d^2 \sigma}{d \Omega \, dE'} = \frac{\alpha_{em}^2}{2MQ^2} \frac{E'}{E} \, L_{\mu \gamma} W^{\mu \gamma}
\]

Leptonic and hadronic tensors have symmetric (S) and anti-symmetric (A) contributions:

\[
\frac{d^2 \sigma}{d \Omega \, dE'} = \frac{\alpha_{em}^2}{2MQ^2} \frac{E'}{E} \left[ L_{\mu \gamma}^{(S)} W^{\mu \gamma(S)} + L_{\mu \gamma}^{(S)} W^{\mu \gamma(S)} + L_{\mu \gamma}^{(A)} W^{\mu \gamma(A)} + L_{\mu \gamma}^{(A)} W^{\mu \gamma(A)} \right]
\]

Leptonic tensor known from QED.
Hadronic tensor describes a-priori unknown hadronic structure, parameterized by:

\[
\begin{align*}
W_{\mu \gamma}^{(A)} & \quad \text{Observable in polarized scattering} & G_1, G_2 \quad \text{(or } g_1, g_2) \\
W_{\mu \gamma}^{(S)} & \quad \text{Observable in unpolarized scattering} & W_1, W_2 \quad \text{(or } F_1, F_2) 
\end{align*}
\]
Structure Functions of the Nucleon

Deep-inelastic scattering in the one-photon exchange approximation can be written as:

\[
\frac{d^2 \sigma}{d \Omega \, dE'} = \frac{\alpha_{em}^2}{2MQ^2} \frac{E'}{E} \, L_{\mu' \nu} \, W^{\mu' \nu}
\]

Leptonic and hadronic tensors have symmetric (S) and anti-symmetric (A) contributions:

\[
\frac{d^2 \sigma}{d \Omega \, dE'} = \frac{\alpha_{em}^2}{2MQ^2} \frac{E'}{E} \left[ L^{(S)}_{\mu' \nu} \, W^{\mu' \nu(S)} + L^{(A)}_{\mu' \nu} \, W^{\mu' \nu(A)} + L^{(S)}_{\mu' \nu} \, W^{\mu' \nu(S)} + L^{(A)}_{\mu' \nu} \, W^{\mu' \nu(A)} \right]
\]

Leptonic tensor known from QED. Hadron tensor describes a-priori unknown hadronic structure, parameterized by:

- \( W^{(A)}_{\mu' \nu} \) Observable in polarized scattering
  \( G_1, G_2 \) (or \( g_1, g_2 \))
- \( W^{(S)}_{\mu' \nu} \) Observable in unpolarized scattering
  \( W_1, W_2 \) (or \( F_1, F_2 \))

Consider unpolarized scattering in the following. Parameterize hadronic structure using \( F_1 \) and \( F_2 \) for which Bjørken predicted scaling:

\[
F_1(x, Q^2) = MW_1(\nu, Q^2) \rightarrow F_1(x) \quad F_2(x, Q^2) = \nu W_2(\nu, Q^2) \rightarrow F_2(x)
\]

\[
\frac{d^2 \sigma}{dx \, dQ^2} = \frac{4 \pi \alpha_{em}^2}{Q^4} \left[ y^2 F_1(x, Q^2) + \left( 1 - y - \frac{M}{2E} \right) \cdot F_2(x, Q^2) \right]
\]
Structure Functions $F_1, F_2$

In naïve Quark-Parton-Model:

$$
F_1 = \frac{1}{2} \sum_f e_f^2 [q(x) + \bar{q}(x)]
\quad \text{Callan-Gross relation}
$$

$$
F_2 = x \sum_f e_f^2 [q(x) + \bar{q}(x)]
\quad F_2 = 2x F_1
$$

Longitudinal ($\sigma_L$) and transverse ($\sigma_T$) virtual-photon contributions:

$$
F_1 = \frac{MK}{4\pi \alpha_{em}} \sigma_T
$$

$$
F_2 = \frac{\nu K (\sigma_L + \sigma_T)}{4\pi \alpha_{em} (1 + Q^2 / 4M^2 x^2)}
$$

Virtual-photon flux

$$
\Gamma = \frac{\alpha_{em} K E'}{2\pi^2 Q^2 E} \frac{1}{1 - \epsilon}
$$

Define ratio $R$ and re-parameterize cross section

$$
R = \frac{\sigma_L}{\sigma_T}
$$

$$
\frac{d^2 \sigma}{dx \, dQ^2} = \frac{4\pi \alpha_{em}^2}{Q^4} \frac{F_2}{x} \times \left[ 1 - y - \frac{Q^2}{4E^2} + \frac{y^2 + Q^2 / E^2}{2(1 + R(x, Q^2))} \right]
$$

Virtual-photon polarization parameter

$$
\epsilon = \frac{4(1 - y) - Q^2 / E^2}{4(1 - y) + 2 \, y^2 + Q^2 / E^2}
$$
Kinematic Plane in x-Q^2

Collider experiments

Fixed target experiments

H1
ZEUS
Fixed Target Experiments:
SLAC, BCDMS
NMC, E665, HERMES

Hera 1
Hera 2 (HERA ~300 GeV)

Why measuring *inclusive DIS cross sections* at Hermes?

Hermes (1996-2005)

- 30 M proton + 28 M deuteron
- ~450 pb⁻¹
- ~460 pb⁻¹

e.g.: compared to NMC

- 3 M proton + 6 M deuteron

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World largest data set on deuteron

\[ F_p^2, \ F_d^2 \]

World data fits

\[ \sigma_p^d, \ \sigma_d^p, \ \sigma_d^p / \sigma_p \]

Gottfried Sum

\[ \int \frac{dx}{x} (F_p^2 - F_n^2) \]

\[ d_v / u_v \]
Binning in $x$ and $Q^2$

- Traditional DIS regime $Q^2 > 1 \text{ GeV}^2$ can be easily separated

**kinematic region**
- $0.006 < x < 0.9$
- $0.1 < y < 0.85$
- $0.2 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$
- $W^2 > 5 \text{ GeV}^2$
- $0.04 \text{ rad} < \Theta < 0.22 \text{ rad}$

**binning**
- 19 $x$ bins
- up to 6 $Q^2$ bins
- Total: 81 bins
Extraction of cross sections

**DIS Yields**

Charge symmetric background
PID flux correction
PID efficiencies
Trigger efficiencies
Luminosity (accidental coincidence)

**Unfolding**

Geometric Acceptance
Detector Smearing
Radiative Corrections
Tracking related effects

**DIS Born cross-section**

**Misalignment** (uncertainty)
Luminosity

- Elastic reference process
- Interaction of beam with shell electrons
  - Electron Beam: Møller scattering \( e^- e^- \rightarrow e^- e^- \)
  - Positron Beam: Bhabha scattering \( e^+ e^- \rightarrow e^+ e^- \), annihilation
  - Coincidence rate \( R_{RL} \) in \( \Delta t = 80\text{ns} \) time resolution window
- Luminosity “constants” \( C_{\text{Lumi}} \) convert coincidence rate into luminosity \( (\text{pb}^{-1}) \)
- Uncertainties of \( \sim 3\% - 8\% \). Acceptance of L. detector depends on e.g.
  - beam conditions
  - magnetic fields

\[
L = \int_{b}^{\infty} \mathcal{L} \, dt = (R_{LR} \cdot \Delta t \cdot R_{L} \cdot R_{R}) \cdot C_{\text{Lumi}} \cdot \frac{A}{Z} \cdot l \cdot \Delta b
\]

- Coincidence rate
- Luminosity constant
- A: nucleons
- Z: shell electrons

\[
\sigma_{\text{DIS}} = \frac{N_{\text{DIS}}}{\int \mathcal{L} \, dt}
\]

DIS yield
Particle identification

TRD → Cherenkov (RICH) → Preshower → Calorimeter

Conditional probabilities for particle hypotheses

\[ \text{PID} = \text{PID}_{\text{det}} - \log_{10} \Phi \]

\[ \Phi = \frac{\phi_h}{\phi_l} = \frac{P(H_h|p, \theta)}{P(H_l|p, \theta)} \]

Relative contributions hadrons and leptons

\[ \text{PID} < 0 \quad \text{Hadrons} \]

\[ \text{PID} > 0 \quad \text{Leptons} \]
**PID efficiency and contamination**

**Lepton sample** identified by: $PID > PID_l$ with $PID_l = 0$
- high efficiencies and small contaminations at same time

**Contamination of lepton sample**

$C = \text{Fractional contribution of hadrons in the lepton sample}$

$$\frac{\int_{PID_l} dPID \ N_h}{\int_{PID_l} dPID \ (N_l + N_h)}$$

**Efficiency lepton identification**

$\varepsilon = \text{Fraction of leptons selected with PID}>0$

$$\frac{\int_{PID_l} dPID \ N_l}{\int dPID \ N_l}$$

**Correction:**

$$N_{\text{cor}} = N_{\text{unc}} \cdot \frac{1 - C(PID_l)}{\varepsilon(PID_l)}$$

Due to correlations between PID detectors, assign uncertainty of full size of correction.
Towards higher $\nu$ (smaller momenta)

- Decreasing **efficiency** ($\geq 98\%$)
- Increasing **contamination** ($\leq 1\%$)

$C$ as a function of $\nu$

$N_{\text{cor}} = N_{\text{unc}} \cdot \frac{1 - C(PID_i)}{\varepsilon(PID_i)}$

Correction small ($\sim 1\%$)
Trigger efficiencies

- Trigger = combination of fast signals
- Select events of specific interest

**DIS trigger (tr21)**

\[ \varepsilon(\text{tr21}) = \varepsilon(H0) \cdot \varepsilon(H1) \cdot \varepsilon(H2) \cdot \varepsilon(\text{CA}) \]

- Trigger efficiencies for each year. Depend on time, momentum, angle.

**Example 2000:**

- \( \varepsilon(H0) \)
- \( \varepsilon(\text{tr21}) \)

\[ w = \frac{1}{\varepsilon} \]

- \( \varepsilon(H1), \varepsilon(H2), \varepsilon(\text{CA}) > 99\% \)
- \( \varepsilon(H0) \sim 97\% \) low!
- Different in top, bot.

- H0 inefficiencies dominate trigger 21 inefficiency → contrib. to top-bot-asym.
QED radiative effects

Feynman diagrams of processes contributing to radiative corrections:

- Initial state radiation
- Final state radiation
- Vacuum polarization
- Vertex correction
Migration matrix

Binning in $x$

Binning in $x-Q^2$

Diagonal elements on migration matrix, measured bin = Born level bin

Migration into acceptance from outside n(i,0), i>0
Unfolding of kinematic bin migration

- Measured Data
- Unfolding
  - Matrix inversion
  - Smearing matrix $S'$
  - Background term $n(i,0)$
- Data on 4π-Born level
- 4π-Born MC
  - Simulation of true cross section
  - No radiative effects
  - No tracking
- Full detector MC
  - Detector material (GEANT4)
  - Radiative effects
  - Tracking
- Background term includes radiative inelastic and elastic events
Bethe-Heitler Cross section

**Bethe-Heitler**: Radiation of real photons associated with elastic interaction of charged particle with the electromagnetic nuclear field

Due to photon radiation, the apparent kinematic variables of Bethe-Heitler events can be indistinguishable from DIS events. → Background to DIS.

### Three cases

- **QED Compton**
  
  *Photon radiation at finite angles*

  \[(1 - y) \sin \theta_{e'} = y \sin \theta_\gamma\]

  → high probability to hit detector frames

- **Initial state radiation (ISR)**
  
  Photon radiation along incoming lepton (lost in the beam pipe)

- **Final state radiation (FSR)**
  
  Photon radiation along outgoing lepton
Bethe-Heitler efficiencies

- Bethe-Heitler efficiencies **extracted from MC** for proton and deuteron

**Proton**

**Deuteron**

- Bethe-Heitler efficiencies are relevant for unfolding
Misalignment

- **Ideal situation**: Perfect alignment of beam and spectrometer
- **In practice**:
  - Top and bottom parts of **spectrometer** displaced
  - **Beam position** differs from nominal position

- Beam misalignment measured by beam monitors
- Analysis of tracks in the top and bottom halves provides information about misalignment of spectrometer

### Misalignment of Beam (1998, 2000) and Spectrometer

<table>
<thead>
<tr>
<th></th>
<th>e⁻</th>
<th>e⁺</th>
<th>top</th>
<th>bot</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-slope (mrad)</td>
<td>-0.014</td>
<td>-0.035</td>
<td>0.44</td>
<td>0.24</td>
</tr>
<tr>
<td>Y-slope (mrad)</td>
<td>-1.200</td>
<td>-0.420</td>
<td>-1.2</td>
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<tr>
<td>X-offset (cm)</td>
<td>0.015</td>
<td>0.017</td>
<td>-0.09</td>
<td>-0.11</td>
</tr>
<tr>
<td>Y-offset (cm)</td>
<td>0.090</td>
<td>0.160</td>
<td>-0.01</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Misalignment

Simulation of misalignment in MC:

- “Misalignment ratio”: Effect of misalignment is in the order of 7%.
- No correction for misalignment but assignment of uncertainty:
  \[ \frac{\sigma_{\text{misaligned}}}{\sigma_{\text{aligned}}} - 1 \]
- Unfolding of the misalignment ratio to Born level
Systematic uncertainties on $\sigma^p, \sigma^d$

- Which systematic uncertainties are assigned.

\[ \delta_{\text{PID}} : \text{PID misidentification} \]
\[ \text{typically } \sim 1\% \]

\[ \delta_{\text{rad.}} : \text{Unc. of BH efficiencies due to misalignment} \]
\[ \lesssim 1\% \]

\[ \delta_{\text{mis}} : \text{Misalignment effect on DIS events} \]
\[ \sim 7\% \]

\[ \delta_{\text{nor}} : \text{Overall normalization unc.: Luminosity} \]
\[ \delta_{\text{nor}}^p = 6.4 \% \quad \delta_{\text{nor}}^d = 6.6 \% \]
Fit to world data of proton DIS cross-sections

Fit with the following features

- Based on the *ALLM functional form* for the $\gamma p$ cross section
  - Regge-motivated, phenomenological approach
  - allows very good description of measured regions
  - constructed so that photoproduction data at $Q^2=0$ can be included

- *Normalization uncertainties* are considered by an accurate method involving a penalty term in $\chi^2$.

- *Fit uncertainties* are determined.
  Covariance matrix provided for the first time.

- *Self-consistent* with respect to the use of $R = \frac{\sigma_L}{\sigma_T}$

- This fit includes *newer data* and covers *2821 data points*. This is more than twice as much as used in ALLM97 (1356 data points).

- Fit results available in *FORTRAN* routine:
  http://www-hermes.desy.de/users/dgabbert/SIGMATOT_PARAM.tgz
Fit to world data of proton DIS cross-sections

• The DIS cross-section in the 1-photon exchange approximation:

\[
\frac{d^2\sigma}{dx \, dQ^2} = \frac{4\pi \alpha^2_{em}}{Q^4} \frac{F_2}{x} \times \left[ 1 - y - \frac{Q^2}{4E^2} + \frac{y^2 + Q^2/E^2}{2(1 + R(x, Q^2))} \right]
\]

for all data sets

• \(F_2\) can be related to the full cross-section \(\sigma = \sigma_L + \sigma_T\)

\[
\sigma_{L+T}(y \, p) = \frac{4\pi \alpha}{Q^2(1-x)} \frac{Q^2 + 4M^2 \, x^2}{Q^2} \frac{F_2}{Q^2} (W^2, Q^2)
\]

• Consistent treatment of \(R\)
Fit to world data of proton DIS cross-sections

- The DIS cross-section in the 1-photon exchange approximation:

\[
\frac{d^{2}\sigma}{dx\,dQ^{2}} = \frac{4\pi\alpha_{em}^{2}}{Q^{4}} \frac{F_{2}}{x} \times \left[ 1 - y - \frac{Q^{2}}{4E^{2}} + \frac{y^{2} + Q^{2}/E^{2}}{2(1+R(x, Q^{2}))} \right]
\]

for all data sets

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\[
\sigma_{L+T}(y, p) = \frac{4\pi\alpha}{Q^{2}(1-x)} \frac{Q^{2} + 4M^{2}x^{2}}{Q^{2}} \frac{F_{2}}{W^{2}, Q^{2}}
\]

- Consistent treatment of \(R\)
$X^2 - \text{Minimization}$

$X^2 - \text{Minimization}$

$$X^2 = \sum_{i}^{n_{\text{max}}} \left( \frac{(\sigma_{i}^{\exp} - \sigma_{i}^{\text{th}})^2}{\delta_{i}^{2 \text{ sta}} + \delta_{i}^{2 \text{ sys}}} \right)$$

General definition
\[ X^2 = \sum_{i}^{n_{\text{max}}} \left( \frac{\sigma_{i}^{\text{exp}} - \sigma_{i}^{\text{th}}}{(1 + \nu_k \delta_{k(i)}^{\text{norm}})} \right)^2 \frac{\delta_{i}^2}{\delta_{i}^{\text{sta}} + \delta_{i}^{\text{sys}}} + \sum_{k} \nu_k^2 \]

- Introduce normalization parameters \( \nu_k \) considered to be normal distributed – implemented by a penalty term.
- The normalization parameters \( \nu_k \) defined in order to perform a re-normalization according to normalization error \( \delta_{k(i)}^{\text{norm}} \).
- The analytic solution of \( \nu_k \) for a fixed set of model parameters can be obtained from \( \frac{d X^2}{d \nu_k} = 0 \), since \( \nu_k \) are independent.
Error propagation

\[ V[\sigma_{L+T}(\mathbf{p}, x, Q^2)] = \sum_{i,j} \text{cov}^{\mathbf{p}}_{i,j} \left( \frac{d \sigma_{L+T}(\mathbf{p}, x, Q^2)}{d p_i} \right) \left( \frac{d \sigma_{L+T}(\mathbf{p}, x, Q^2)}{d p_j} \right) \]

\[ V \quad \text{variance} \]

\[ \mathbf{p} \quad \text{parameter vector} \]

\[ \text{cov}^{\mathbf{p}}_{i,j} \quad \text{covariance matrix for } \mathbf{p} \]
## $F_2$ fit results (GD08)

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Exp</th>
<th>n</th>
<th>$\chi^2/n$</th>
<th>$\delta_k^{\text{nor}}$</th>
<th>$\nu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>SLAC-E49a</td>
<td>98</td>
<td>0.51</td>
<td>2.1</td>
<td>0.06</td>
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<tr>
<td>2.</td>
<td>SLAC-E49b</td>
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<td>SLAC-E89b</td>
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<td>7.</td>
<td>NMC 90 GeV</td>
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<td>10.</td>
<td>NMC 280 GeV</td>
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<td>14.</td>
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<td>16.</td>
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<tr>
<td>31.</td>
<td>this analysis, HERMES</td>
<td>81</td>
<td>0.40</td>
<td>6.4</td>
<td>0.67</td>
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Total: $\chi/n = 0.93$
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</table>

Total: $\chi/n = 0.93$

Low value of $\chi^2/n$ reflects conservative assignment of uncertainties: misalignment, overall normalization.
$F_2$ fit results (GD08)
Results on $F_2^p$

Proton

$F_2 \cdot c$

<table>
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<tr>
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<th>c</th>
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<tr>
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$Q^2$, GeV$^2$
Results on $F_2^p$

Proton

$F_2 \cdot c$

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</table>

$Q^2$, GeV$^2$
Results on $F_2^p$

Proton

$F_2^p$ vs $Q^2$, GeV$^2$

GD08
GD07
ALLM97
SMC
Results on $F_2^d$

Deuterion

$F_2 \cdot c$

- SLAC
- BCDMS
- JLAB
- NMC
- this analysis
- E665

<table>
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<td>$1.6^{+0}_{-26}$</td>
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<td>$1.6^{+0}_{-28}$</td>
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$Q^2$, GeV$^2$
Results on $F_2^d$

Deuteron
Results on $F_2^d$

Deuteron

$F_2^d$ vs. $Q^2$, GeV$^2$

SMC
Why measuring *inclusive DIS cross sections* at Hermes?

\[ F_2^p, F_2^d \]

World data fits
\[ \sigma_{p,d}^{p,d} / \sigma_p \]

Gottfried Sum
\[ \int \frac{dx}{x} \left( F_2^p - F_2^n \right) \]

\[ d_v / u_v \]
Basic nucleon structure from sum rules

Quark-Parton Model

Adler sum rule

\[ \int \frac{dx}{x} (F_2^p - F_2^n) = \int \frac{dx}{x} (F_2^p - F_2^n) = 2 \int dx (u_v - d_v) = 2 \]

Gross-Llewellyn Smith sum rule

\[ \int dx (F_3^p + F_3^n) = \int dx (F_3^p + F_3^n) = 2 \int dx (u_v + d_v) = 6 \]

Gottfried sum rule

\[ \ldots \ldots \]
Basic nucleon structure from sum rules

Quark-Parton Model

Adler sum rule
\[ \int dx \frac{1}{x} (F_2^p - F_2^n) = \int dx \frac{1}{x} (F_2^\nu - F_2^{\overline{\nu}}) = 2 \int dx (u_v - d_v) = 2 \]

Gross-Llewellyn Smith sum rule
\[ \int dx (F_3^p + F_3^n) = \int dx (F_3^\nu + F_3^{\overline{\nu}}) = 2 \int dx (u_v + d_v) = 6 \]

Gottfried sum rule
\[ \int dx \frac{1}{x} (F_2^e,\mu^p - F_2^e,\mu^n) = \frac{1}{3} \int dx (u_v - d_v) + \frac{2}{3} \int dx (\overline{u} - \overline{d}) = \frac{1}{3} \]

- Gottfried sum rule (charged lepton scattering) → Sensitive to

- Difference between u and d valence quarks
- **Sea quark flavor symmetry** / asymmetry?
Sea quark asymmetry

Measurements, e.g.:

DIS data: SLAC, BCDMS, NMC, HERMES
Drell-Yan data: E288, (E772), NA51, E866

E.g.: NMC($Q^2=4$ GeV$^2$): $I_G(0.004, 0.8) = 0.236\pm0.008$
Extrapolation: $I_G(0,1) = 0.258\pm0.017$
→ Significant violation of Gottfried sum rule
Sea flavor asymmetry $\bar{u} \neq \bar{d}$. $I_G(0,1) < 1/3$: excess of $\bar{d}$ quarks over $\bar{u}$ quarks.

$\bar{d}$ quark excess confirmed in Drell-Yan and semi-inclusive analysis:
The Gottfried Integral

\[ I_G(x_{\text{min}}, x_{\text{max}}) = \int_{x_{\text{min}}}^{x_{\text{max}}} \frac{(F_2^p(x) - F_2^n(x))}{x} \, dx = \int_{x_{\text{min}}}^{x_{\text{max}}} 2(F_2^p - F_2^d) \, dx / x \]

\[ = \int_{x_{\text{min}}}^{x_{\text{max}}} 2 F_2^p \left( 1 - \frac{F_2^d}{F_2^p} \right) \, dx / x \]

Evaluation of the **measured** Gottfried Integral:
- GD 08  
- Fit to \( \sigma^d/\sigma^p \)

Evaluation of the **leading twist (LT)** Gottfried integral
- CTEQ6L  
- Fit to \( \sigma^d/\sigma^p \) (LT)
Fits to $\sigma^d/\sigma^p$

\[
\frac{\sigma^d}{\sigma^p} \approx \frac{F_2^d}{F_2^p} \left( 1 - \frac{1 - \epsilon}{(1 + \bar{R}(x))(1 + \epsilon \bar{R}(x))} \Delta R(x) \right)
\]

Relation of cross-section ratio to $F_2$-ratio

\[
\frac{F_2^d}{F_2^p} \approx \frac{F_2^{d,LT}}{F_2^{p,LT}}(x, Q^2)(1 + \frac{C^d(x) - C^p(x)}{Q^2})
\]

Higher twist effects

\[
\frac{F_2^{d,LT}}{F_2^{p,LT}}(x, Q^2) \approx b_1(x) + b_2(x) \ln Q^2
\]

$Q^2$ evolution

Parameterization

\[
\frac{\sigma^d}{\sigma^p} \approx \frac{F_2^d}{F_2^p} \left( b_1(x) + b_2(x) \ln Q^2 \right) \left( 1 + \frac{C^d(x) - C^p(x)}{Q^2} \right) \left( 1 - \frac{1 - \epsilon}{(1 + \bar{R}(x))(1 + \epsilon \bar{R}(x))} \Delta R(x) \right)
\]

- **4-parameter Fit** in each $x$ bin
  based on world data from **NMC, SLAC, BCDMS, HERMES**
World data on $\sigma^d/\sigma^p$
4 Parameters from Fit of $\sigma^d/\sigma^p$ to world data

- $F_2^d/F_2^p(Q^2=1)$: approaching unity for small $x$.
- $d (F_2^d/F_2^p) / d\ln Q^2$: tendency to negative slopes at high $x$.
- $C_d - C_p$: significant only at $x>0.2$
- $R_d - R_p$: Consistent with zero
Evaluation of the Gottfried integral

<table>
<thead>
<tr>
<th>$Q^2$</th>
<th>$x$ - range</th>
<th>$I_G^{\text{meas.}}$</th>
<th>$I_G^{LT}$</th>
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<td>0.208±0.016</td>
<td>0.236±0.006</td>
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<td>4 GeV$^2$</td>
<td>0.006-0.9</td>
<td>0.228±0.006</td>
<td>0.228±0.006</td>
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<td>10 GeV$^2$</td>
<td>0.006-0.9</td>
<td>0.228±0.013</td>
<td>0.228±0.012</td>
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</table>
Extraction of $d_v / u_v$

$$\frac{F_2^d}{F_2^p} = \frac{1}{2} \left( 1 + \frac{F_2^n}{F_2^p} \right) \approx \frac{1}{2} \left( 1 + \frac{4 \frac{d_v}{u_v} + 1 + S_1}{4 + \frac{d_v}{u_v} + S_2} \right) \approx \frac{5}{2} \cdot \frac{1 + \frac{d_v}{u_v}}{4 + \frac{d_v}{u_v}}$$

$S_{1,2} = 0$

$S_1 = \frac{2}{u_v} (u_s + 4d_s + s_s)$

$S_2 = \frac{2}{u_v} (4u_s + d_s + s_s)$

- $S_1$ and $S_2$ taken from CTEQ6L
- Impact of $S_1$ and $S_2$ negligible at $x > 0.35$

- Comparison of $d_v/u_v$ with CTEQ6L (LO) result reveals compatibility.
Summary

• First measurement of $F_2^p$ and $F_2^d$ at Hermes.

• Fit of the proton DIS cross section based on the ALLM functional form
  - Larger data set, 2821 data points, incl. Hermes
  - Self-consistent with respect to R
  - Normalization uncertainties taken into account
  - Covariance matrix provided

• Fit of the cross section ratio $\sigma^d/\sigma^p$
  - Extraction of the Gottfried integral
  - Compatibility with the NMC result
  - Indicates violation of Gottfried sum rule
  - No indication for $Q^2$ dependence found