gmc_trans
a Monte Carlo generator for transverse-momentum-dependent distribution and fragmentation functions

A. Bacchetta, U. Elschenbroich, N. Makins, G. Schnell, R. Seidl

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Outline

- Motivation & Basics
- Details of the MC generator
  - Gaussian Ansatz
  - Positivity limits
  - Event generation
- Some results
  - Tuning of transverse momentum dependence
  - (un)weighted Sivers and Collins amplitudes
- Implemented models
- Going beyond Collins and Sivers asymmetries
Monte Carlo Simulations are an indispensable tool in modern nuclear and particle physics experiments.

Various “physics generators” exist for the various fields (e.g., PYTHIA, LEPTO, AROMA etc.)

Used for predictions, for the understanding of the experiment, and also for the “correction” of data (e.g., acceptance effects, background processes etc.)

No generator was available for transverse-momentum dependence of distribution and fragmentation functions.
Initial goals for gmc_trans

- physics generator for SIDIS pion production
- include transverse-momentum dependence, in particular simulate Collins and Sivers effects
- be fast
- allow comparison of input model and reconstructed amplitudes
- to be used with standard HERMES Monte Carlo
- be extendable (e.g., open for new models)
Basic workings

- use cross section that can (almost) be calculated analytically
- use Gaussian Ansatz for all transverse-momentum dependencies of DFs and FFs
- unpolarized DFs (as well as helicity distribution) and FFs from fits/parametrizations (e.g., Kretzer FFs etc.)
- “polarized” DFs and FFs either related to unpolarized ones (e.g., saturation of Soffer bound for transversity) or some parametrizations used
SIDIS Cross Section incl. TMDs

\[ d\sigma_{UT} \equiv d\sigma_{UT}^{Collins} \cdot \sin(\phi + \phi_S) + d\sigma_{UT}^{Sivers} \cdot \sin(\phi - \phi_S) \]

\[ d\sigma_{UT}^{Collins}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sx y^2} B(y) \sum_q e_q^2 \mathcal{I} \left[ \left( \frac{k_T \cdot \hat{P}_{h\perp}}{M_h} \right) \cdot h_1^q \overset{\perp}{H_1^q} \right] \]

\[ d\sigma_{UT}^{Sivers}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sx y^2} A(y) \sum_q e_q^2 \mathcal{I} \left[ \left( \frac{p_T \cdot \hat{P}_{h\perp}}{M_N} \right) \cdot f_{1T}^\perp D_1^q \right] \]

\[ d\sigma_{UU}(x, y, z, \phi_S, P_{h\perp}) \equiv \frac{2\alpha^2}{sx y^2} A(y) \sum_q e_q^2 \mathcal{I} \left[ f_1^q D_1^q \right] \]

where

\[ \mathcal{I} \left[ \mathcal{W} f D \right] \equiv \int d^2 p_T d^2 k_T \delta^{(2)} \left( p_T - \frac{P_{h\perp}}{z} - k_T \right) \left[ \mathcal{W} f(x, p_T) D(z, k_T) \right] \]
Gaussian Ansatz

- want to deconvolve convolution integral over transverse momenta

- **easy Ansatz**: Gaussian dependencies of DFs and FFs on intrinsic (quark) transverse momentum:

\[ \mathcal{I}[f_1(x, p^2_T)D_1(z, z^2k^2_T)] = f_1(x) \cdot D_1(z) \cdot \frac{R^2}{\pi z^2} \cdot e^{-R^2 \frac{p^2_T}{z^2}} \]

with \( f_1(x, p^2_T) = f_1(x) \frac{1}{\pi \langle p^2_T \rangle} e^{-\frac{p^2_T}{\langle p^2_T \rangle}} \)

\[ \frac{1}{R^2} = \langle k^2_T \rangle + \langle p^2_T \rangle = \frac{\langle p^2_{h \perp} \rangle}{z^2} \]

(similar: \( D_1(z, z^2k^2_T) \))

**Caution**: different notations for intrinsic transverse momentum exist!
**Positivity Constraints**

- DFs (FFs) have to fulfill various positivity constraints (resulting cross section must not be negative!)
- Based on probability considerations can derive positivity limits for leading-twist functions: 

→ Transversity: e.g., Soffer bound

→ Sivers and Collins functions: e.g., loose bounds:

\[
\frac{|p_T|}{2M_N} f_{1T}^\perp (x, p_T^2) \equiv f_{1T}^{(1/2)} (x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2)
\]

\[
\frac{|k_T|}{2M_h} H_1^\perp (z, z^2 k_T^2) \equiv H_1^{(1/2)} (z, z^2 k_T^2) \leq \frac{1}{2} D_1(z, z^2 k_T^2)
\]
Positivity and the Gaussian Ansatz

\[ \frac{|p_T|}{2M_N} f_{1T}^\perp(x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2) \]

with \( f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \)

\[ f_{1T}^\perp(x, p_T^2) = f_{1T}^\perp(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \]

\[ |p_T| f_{1T}^\perp(x) \leq M_N f_1(x) \]
Positivity and the Gaussian Ansatz

\[
\left| p_T \right| \frac{f_{1T}^\perp(x, p^2_T)}{2M_N} \leq \frac{1}{2} f_1(x, p^2_T)
\]

with
\[
f_1(x, p^2_T) = f_1(x) \frac{1}{\pi \langle p^2_T \rangle} e^{-\frac{p^2_T}{\langle p^2_T \rangle}}
\]

\[
f_{1T}^\perp(x, p^2_T) = f_{1T}^\perp(x) \frac{1}{\pi \langle p^2_T \rangle} e^{-\frac{p^2_T}{\langle p^2_T \rangle}}
\]

\[
\left| p_T \right| f_{1T}^\perp(x) \leq M_N f_1(x)
\]

Problem for non-zero Sivers function!
Modify Gaussian width

\[ f_{1T}^\perp(x, p_T^2) = f_{1T}^\perp(x) \frac{1}{(1 - C)\pi\langle p_T^2 \rangle} e^{-\frac{p_T^2}{(1 - C)\langle p_T^2 \rangle}} \]

\[ \rightarrow \text{positivity limit:} \]

\[ f_{1T}^\perp(x) \frac{|p_T|}{2M_N} \frac{1}{\pi(1 - C)\langle p_T^2 \rangle} e^{-\frac{p_T^2}{(1 - C)\langle p_T^2 \rangle}} \leq 1/2 f_1(x) \frac{1}{\pi\langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \]

\[ \frac{|p_T|}{1 - C} e^{-\frac{C}{1 - C} \frac{p_T^2}{\langle p_T^2 \rangle}} \leq M_N \frac{f_1(x)}{f_{1T}^\perp(x)} \]
New positivity limit

\[ \frac{|p_T|}{1 - C} e^{-\frac{C}{1-C} \frac{p_T^2}{\langle p_T^2 \rangle}} \leq M_N \frac{f_1(x)}{f_{1T}(x)} \]

Minimum at \( p_T = \sqrt{\frac{\langle p_T^2 \rangle}{2C}} \)

thus \( \frac{f_{1T}^\perp(x)}{f_1(x)} \leq M_N \sqrt{\frac{2eC(1-C)}{\langle p_T^2 \rangle}} \)

or \( \frac{f_{1T}^{(1/2)}(x)}{f_1(x)} \leq \frac{1}{2} \sqrt{\frac{e\pi C}{2}} (1 - C) \leq 0.4 \)
\[
\sum_{q} \frac{e_{q}^{2}}{4\pi (M_{E}xyz)^{2}} [X_{UU} + |S_{T}|X_{SIV} \sin(\phi_{h} - \phi_{s}) + |S_{T}|X_{COL} \sin(\phi_{h} + \phi_{s})]
\]

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

\[
X_{UU} = R^{2} e^{-R^{2} P_{h\perp} / z^{2}} \left(1 - y + \frac{y^{2}}{2}\right) f_{1}(x) \cdot D_{1}(z)
\]

\[
X_{COL} = + \frac{|P_{h\perp}|}{M_{\pi z}} \frac{(1 - C)\langle k_{T}^{2} \rangle}{\left[\langle p_{T}^{2} \rangle + (1 - C)\langle k_{T}^{2} \rangle \right]^{2}} \exp \left[-\frac{P_{h\perp} / z^{2}}{\langle p_{T}^{2} \rangle + (1 - C)\langle k_{T}^{2} \rangle} \right]
\]
\[
\times (1 - y) \cdot h_{1}(x) \cdot H_{1}^{\perp}(z)
\]

\[
X_{SIV} = - \frac{|P_{h\perp}|}{M_{p z}} \frac{(1 - C')\langle p_{T}^{2} \rangle}{\left[\langle k_{T}^{2} \rangle + (1 - C')\langle p_{T}^{2} \rangle \right]^{2}} \exp \left[-\frac{P_{h\perp} / z^{2}}{\langle k_{T}^{2} \rangle + (1 - C')\langle p_{T}^{2} \rangle} \right]
\]
\[
\times \left(1 - y + \frac{y^{2}}{2}\right) f_{1T}^{\perp}(x) \cdot D_{1}(z)
\]
Sivers (azimuthal) moments

use cross section expressions to evaluate azimuthal moments:

\[- \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{\sqrt{(1 - C)\langle p_T^2 \rangle}}{\sqrt{(1 - C)\langle p_T^2 \rangle + \langle k_T^2 \rangle}} \frac{A(y)\frac{1}{x y^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{x y^2} \sum e_q^2 f_1(x)D_1(z)} \]

\[- \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{M_N\sqrt{\pi}}{2\sqrt{(1 - C)\langle p_T^2 \rangle + \langle k_T^2 \rangle}} \frac{A(y)\frac{1}{x y^2} \sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{x y^2} \sum e_q^2 f_1(x)D_1(z)} \]

\[- \langle \frac{|P_{h \perp}|}{z M_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{2\sqrt{(1 - C)\langle p_T^2 \rangle}}{M_N\sqrt{\pi}} \frac{A(y)\frac{1}{x y^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x)D_1(z)}{A(y)\frac{1}{x y^2} \sum e_q^2 f_1(x)D_1(z)} \]

\[- \langle \frac{|P_{h \perp}|}{z M_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{A(y)\frac{1}{x y^2} \sum e_q^2 f_{1T}^{\perp(1)}(x)D_1(z)}{A(y)\frac{1}{x y^2} \sum e_q^2 f_1(x)D_1(z)} \]

model-dependence on transverse momenta “swallowed” by $p_T^2$-moment of Sivers fct.: $f_{1T}^{\perp(1)}$
Sivers (azimuthal) moments

use cross section expressions to evaluate azimuthal moments:

\[
- \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{\sqrt{(1 - C) \langle p_T^2 \rangle}}{\sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp (1/2)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}
\]

\[
- \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{M_N \sqrt{\pi}}{2 \sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp (1)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}
\]

\[
- \frac{|P_{h\perp}|}{z M_N} \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{2 \sqrt{(1 - C) \langle p_T^2 \rangle}}{M_N \sqrt{\pi}} \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp (1/2)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}
\]

\[
- \frac{|P_{h\perp}|}{z M_N} \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp (1)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}
\]

(similar for Collins moments)
Monte Carlo event generation

- need to generate events according to cross section:
  - throw flavor of struck quark according to integrated (unpolarized) cross section for each quark flavor
  - throw \((x, Q^2, z)\) according to unpolarized cross section
  - throw pion’s transverse momentum \(P^2_{h\perp}\) according to Gaussian Ansatz
  - generate azimuthal angles \((\phi, \phi_S)\) according to polarized cross section
- cross section should be positive automatically if positivity constraints on DFs and FFs are fulfilled, but better check again
Some (more) details

- event generation by accept-or-reject method:
  - first throw flat in \((x, Q^2, z)\), e.g., get value \(r_1\)
  - second random number \(r_2\) determines accept/reject status:
    - \(r_2\) above curve: reject
    - \(r_2\) below curve: accept
  - have to know maximum of \(f(x)\), i.e., of the cross section (checked at beginning)
  - can gain some speed by not throwing flat in, e.g., \(Q^2\), but according to global behaviour
Some Results
Tuning the Gaussians in gmc_trans

- **constant Gaussian widths, i.e., no dependence on x or z:**
  \[
  \langle p_T \rangle = 0.44 \\
  \langle K_T \rangle = 0.44
  \]

- **tune to data integrated over whole kinematic range**
Tuning the Gaussians in `gmc_trans`

\[ \langle p_T \rangle = 0.38 \]
\[ \langle K_T \rangle = 0.38 \]
\[ \langle p_T^2 \rangle = \langle K_T^2 \rangle = 0.18 \text{GeV}^2 \quad (\langle |p_T| \rangle = \langle |K_T| \rangle = 0.38 \text{GeV}) \]

where: \[ \langle K_T^2 \rangle = z^2 \langle k_T^2 \rangle \]
Tuning the Gaussians in gmc_trans

in general: \[ \langle P^2_{h\perp}(x, z) \rangle = z^2 \langle p^2_T(x) \rangle + \langle K^2_T(z) \rangle \]

so far: \[ \langle P^2_{h\perp}(z) \rangle = z^2 \langle p^2_T \rangle + \langle K^2_T \rangle \]

constant!
Tuning the Gaussians in gmc_trans

so far: \[ \langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle \]

\[ \langle p_T \rangle = 0.38 \]
\[ \langle K_T \rangle = 0.38 \]

\[ \langle p_T^2 \rangle \simeq 0.185 \]
\[ \langle K_T^2 \rangle \simeq 0.185 \]
Tuning the Gaussians in gmc_trans

now: \( \langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2(z) \rangle \)

- \( P_{h\perp}^2 \)
- \( p_T^2 z^2 = 0.144362 z^2 \)
- \( K_T^2 = 0.422458 z^{0.536321} (1-z)^{0.365594} \)

z-dependent!

tuned to HERMES data in acceptance

“Hashi set”
Tuning the Gaussians in gmc_trans

\[ \langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2(z) \rangle \]

z-dependent!

“Hashi set”
\[2 \sin(\phi_{\pi^0})_{\text{ur}}\]

\[\begin{align*}
\pi^+ & : \delta u(x) = 0.7 \cdot \Delta u(x) & f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) & H_{1,\text{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\text{fav}}^{\perp}(z) \\
\pi^- & : \delta d(x) = 0.7 \cdot \Delta d(x) & f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) & H_{1,\text{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\text{dis}}^{\perp}(z) \\
\end{align*}\]

\[\begin{align*}
\delta q(x) & = 0.3 \cdot \Delta q(x) & f_{1T}^{\perp q}(x) = 0.0 & q = \bar{u}, d, s, \bar{s} \\
C_S & = C_C = 0.25
\end{align*}\]
Comparison for weighted moments

Not so good news for weighted moments

Gunar Schnell, Universiteit Gent
Where to go from here?
Models for integrated DFs and FF

- “usual PDFs” $f_1, g_1$ from PEPSI library
- $D_1$ from KKP or Kretzer
- $h_1$ can be
  - $= g_1$
  - Soffer bound
  - Leader parametrization
Models for Sivers and Collins Fcts

- **Sivers function** $f_{1T}^\perp$
  - $f_{1T}^\perp(x) \sim f_1(x)$
  - $f_{1T}^\perp(x) \sim g_1(x)$

- Boglione-Mulders parametrization
  - $f_{1T}^{\perp(1)}(x) \sim f_1(x)$

- **Collins function:** $H_1^\perp$
  - $H_1^\perp(z) \sim D_1(z)$

- Boglione-Mulders

- Leader
  - $H_1^{\perp(1)}(z) \sim D_1(z)$
Models for Sivers and Collins Fcts

- **Sivers function** $f_{1T}^\perp$
  - $f_{1T}^\perp(x) \sim f_1(x)$
  - $f_{1T}^\perp(z) \sim g_1(z)$

- Boglione-Mulders parametrization
  - $f_{1T}^{(1)}(x) \sim f_1(x)$

- **Collins function**
  - $H_1^\perp(z) \sim D_1(z)$
  - $H_1^{(1)}(z) \sim D_1(z)$

**obviously want more (new) fits/parametrizations**
certainly would like to model all TMDs, e.g., Boer-Mulders function, to get full cross section

even go to subleading-twist, e.g., Cahn effect

first attempts to implement those have been made

leading twist -- “straight forward” (just a few more convolution integrals)

subleading twist -- “hmmmm...”

biggest problem there: positivity limits don’t exist on DF and FF level
Status of unpolarized cross section

- almost implemented:
  - Boer-Mulders effect
  - Cahn effect
  - can adjust Gaussian width, kinematic dependencies and normalization
  - generated values (cross-section and unweighted moments) are available for end-user, however, weighted moments not available
Status of polarized cross section

hardly implemented:

- twist-3 AUT $\sin \phi_S$ term (involves transversity)
  - can partially adjust Gaussian width (involves 2 terms - only the one involving the Collins function is adjustable), kinematic dependencies and normalization

- generated values: cross-section is available for end-user, however, moments are not available
\[ \sin \phi_s \text{ - term in } A_{\text{UT}} \]

\[ - \mathcal{I} \left[ \frac{k_T \cdot p_T}{2 M M_h} \left( x h_T H_1^\perp - x h_T^\perp H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} + \frac{M_h}{M} f_{1T} \frac{\tilde{D}^\perp}{z} \right) \right] \]

\[ - h_1 + h_T - \frac{p_T^2}{2 M^2 x} h_{1T}^\perp \]

\[ h_1 - h_T^\perp - \frac{p_T^2}{2 M^2 x} h_{1T}^\perp \]

\[ \left[ -2 h_1 + (\tilde{h}_T - \tilde{h}_T^\perp) \right] \]
$\sin \phi_S$ - term in $A_{UT}$

\[2(2-y)\sqrt{1-y} \frac{M}{Q} \int \left[ \frac{k_T \cdot p_T}{2M M_h} [2h_1 + x(\tilde{h}_T - \tilde{h}_T)] \right] \left[ H_{1}^\perp - \frac{M_h}{zM} h_1 \tilde{H} \right]
\]

convolution integral and DF & FF known/implemented

twist-3 FF (Koike?) (turned off in gmc_trans)
Cahn Effect

- similar to twist-3 AUT term
- reduced to product of $f_1$ and $D_1$
- neglect any interaction-dependent terms
- get $f_1$ and $D_1$ from PDF/FF library

however,

- need additional scaling factor, and
- need different Gaussian width than normal $f_1$ and $D_1$
The Twist-3 Problems

- need scaling factors, even though Mulders&Tangerman etc. would suggest a normalization that is fixed by involved PDFs and FFs

- same is true for Gaussian widths: need different ones than for “normal” $f_1$ and $D_1$ (or transversity and Collins FF)

- therefore: intrinsically inconsistent treatment -- failure of Gaussian Ansatz?

- in general: encountered severe positivity violations (in particular with Cahn effect)
Summary

- **gmc_trans** is a working MC generator for TMDs in SIDIS (pion production)
- Based on Gaussian Ansatz for transverse momentum dependencies
- Collins and Sivers effect implemented
- Z-dependence of Gaussian widths tuned to HERMES data
- Implementation of other (partially subleading-twist) terms not straight-forward
Outlook / Wishlist

- finish Boer-Mulders implementation
- implement newest results from fits and model calculations on transversity, Sivers & Collins functions
- implement Kaons and neutron target
  
  ➡  comparison with HERMES and COMPASS data possible

- add radiative corrections (RADGEN)
- solve twist-3 problems
- gmc_trans for 2-hadron production