Transverse Spin Physics at HERMES

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Transverse Single-Spin Asymmetries in pp Collisions

\[ p^+ p \rightarrow \pi X \]

E704 $\sqrt{s} = 20$ GeV

STAR $\sqrt{s} = 200$ GeV

left-right-asymmetry w.r.t. incoming proton’s spin

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SSAs persist even at high energies! (despite being suppressed in pQCD)

- asymmetry in quark fragmentation (Collins)
- asymmetry in quark distribution (Sivers)
- subleading-twist effects
27.5 GeV $e^+/e^-$ beam of HERA

- forward-acceptance spectrometer
  $\Rightarrow\ 40\text{mrad}<\theta<220\text{mrad}$
- high lepton ID efficiency and purity
- excellent hadron ID thanks to dual-radiator RICH
atomic beam source
⇒ pure gas target, no dilution
transversely pol. hydrogen
polarization \( \sim 75\% \)
90s flipping time ⇒ small systematics
Transverse Spin of the Nucleon
Quark Distribution Functions

$$f_1^q = \bullet$$  \hspace{2cm}  $$g_1^q = \bullet \rightarrow \bullet$$  \hspace{2cm}  $$h_1^q = \bullet \rightarrow \bullet$$

$$\downarrow \downarrow \downarrow$$

Unpolarized quarks and nucleons  \hspace{2cm}  Longitudinally polarized quarks and nucleons  \hspace{2cm}  Transversely polarized quarks and nucleons

$$f_1^q(x)$$: spin averaged  \hspace{2cm}  $$g_1^q(x)$$: helicity difference  \hspace{2cm}  $$h_1^q(x)$$: transversity

(Well known)  \hspace{2cm}  (Known)  \hspace{2cm}  (Hardly known!)

$$\Rightarrow$$ Vector Charge  \hspace{2cm}  $$\Rightarrow$$ Axial Charge  \hspace{2cm}  $$\Rightarrow$$ Tensor Charge

$$\langle PS | \bar{\Psi} \gamma^\mu \Psi | PS \rangle = \int dx (f_1^q(x) - f_1^\bar{q}(x))$$  \hspace{2cm}  $$\langle PS | \bar{\Psi} \gamma^\mu \gamma_5 \Psi | PS \rangle = \int dx (g_1^q(x) + g_1^\bar{q}(x))$$  \hspace{2cm}  $$\langle PS | \bar{\Psi} \sigma^{\mu\nu} \gamma_5 \Psi | PS \rangle = \int dx (h_1^q(x) - h_1^\bar{q}(x))$$
Quark Distribution Functions

Unpolarized quarks and nucleons

\[ f_1^q(x) : \text{spin averaged (well known)} \]

\[ \Rightarrow \text{Vector Charge} \]

\[ \langle PS | \bar{\Psi} \gamma^\mu \Psi | PS \rangle = \int dx (f_1^q(x) - f_1^\bar{q}(x)) \]

Longitudinally polarized quarks and nucleons

\[ g_1^q(x) : \text{helicity difference (known)} \]

\[ \Rightarrow \text{Axial Charge} \]

\[ \langle PS | \bar{\Psi} \gamma^\mu \gamma_5 \Psi | PS \rangle = \int dx (g_1^q(x) + g_1^\bar{q}(x)) \]

Transversely polarized quarks and nucleons

\[ h_1^q(x) : \text{transversity (hardly known!)} \]

\[ \Rightarrow \text{Tensor Charge} \]

\[ \langle PS | \bar{\Psi} \sigma^{\mu\nu} \gamma_5 \Psi | PS \rangle = \int dx (h_1^q(x) - h_1^\bar{q}(x)) \]
Quark Distribution Functions

Unpolarized quarks and nucleons

\[ f_1^q(x) : \text{spin averaged} \]
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Transversely polarized quarks and nucleons

\[ h_1^q(x) : \text{transversity (hardly known!)} \]

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\[ \langle PS| \bar{\Psi} \sigma^{\mu\nu} \gamma_5 \Psi |PS\rangle = \int dx (h_1^q(x) - h_1^{\bar{q}}(x)) \]

CHIRAL-ODD!
transverse spin eigenstates related to helicity eigenstates
via \(|⊥\perp⟩\) = \(\frac{1}{2}(|+⟩ ± \imath |−⟩)\) \(\implies\) transversity \((⟨⊥|\hat{O}|⊥⟩ − ⟨\perp|\hat{O}|\perp⟩)\)
flips helicity of quark and nucleon \(\Rightarrow h^q_{1}\) chiral odd

\(\implies\) No Access In Inclusive DIS!
Transversity Distribution

- transverse spin eigenstates related to helicity eigenstates via \( |\perp T\rangle = \frac{1}{2} (|+\rangle \pm i|\mp\rangle) \iff \) transversity \( \langle \perp |\hat{O}|\perp \rangle - \langle T |\hat{O}|T \rangle \) flips helicity of quark and nucleon \( \Rightarrow h_1^q \) chiral odd

\[ \iff \] No Access In Inclusive DIS!

- no “gluon transversity”

\[ \Rightarrow \) different \( Q^2 \)-evolution than for \( f_1^q(x) \) and \( g_1^q(x) \)
Transversity Distribution

- Transverse spin eigenstates related to helicity eigenstates via $|⊥^T⟩ = \frac{1}{2} (|+⟩ ± i|−⟩) \implies$ transversity $(⟨⊥|\hat{O}|⊥⟩ − ⟨T|\hat{O}|T⟩)$ flips helicity of quark and nucleon $⇒ h_q^1$ chiral odd

  $⇒$ No Access In Inclusive DIS!

- No “gluon transversity”
  $⇒$ different $Q^2$-evolution than for $f_q^1(x)$ and $g_q^1(x)$

- Non-relativistic quarks: $h_q^1(x) = g_q^1(x)$
  $⇒ h_q^1$ probes relativistic nature of quark dynamics
Transversity Distribution

- transverse spin eigenstates related to helicity eigenstates via \( |\perp T\rangle = \frac{1}{2} (|+\rangle \pm i |\rangle - \rangle) \Rightarrow \) transversity \((\langle \perp |\hat{O}|\perp \rangle - \langle T |\hat{O}| T \rangle)\) flips helicity of quark and nucleon \( \Rightarrow h_1^q \) chiral odd

  \[ \rightarrow \quad \text{No Access In Inclusive DIS!} \]

- no "gluon transversity"
  \( \Rightarrow \) different \( Q^2 \)-evolution than for \( f_1^q(x) \) and \( g_1^q(x) \)

- Non-relativistic quarks: \( h_1^q(x) = g_1^q(x) \)
  \( \Rightarrow h_1^q \) probes \textit{relativistic nature} of quark dynamics

- positivity bounds: \( |h_1^q(x)| \leq f_1^q(x) \)
  \[ |h_1^q(x)| \leq \frac{1}{2} [f_1^q(x) + g_1^q(x)] \]
transverse spin eigenstates related to helicity eigenstates via $|\perp T\rangle = \frac{1}{2}(|+\rangle \pm \imath |-\rangle)$ $\implies$ transversity $(\langle \perp |\hat{O}|\perp\rangle - \langle T |\hat{O}|T\rangle)$ flips helicity of quark and nucleon $\Rightarrow h_1^q$ chiral odd

$\leftrightarrow$ No Access In Inclusive DIS!

no “gluon transversity”
$\Rightarrow$ different $Q^2$-evolution than for $f_1^q(x)$ and $g_1^q(x)$

Non-relativistic quarks: $h_1^q(x) = g_1^q(x)$
$\Rightarrow h_1^q$ probes relativistic nature of quark dynamics

positivity bounds: $|h_1^q(x)| \leq f_1^q(x)$
$|h_1^q(x)| \leq \frac{1}{2}[f_1^q(x) + g_1^q(x)]$

first moment $\rightarrow$ tensor charge calculable in lattice QCD
How can one measure transversity?

Need another chiral-odd object!
How can one measure transversity?
Need another chiral-odd object!
⇒ Semi-Inclusive DIS

\[ \sigma^{ep \to ehX} = \sum_q h_1^q \otimes \sigma^{eq \to eq} \otimes FF^{q \to h} \]

[Diagram showing the process of measuring transversity with Semi-Inclusive DIS]
How can one measure transversity?

Need another chiral-odd object!

⇒ Semi-Inclusive DIS

\[
\sigma^{ep \to ehX} = \sum_q h_1^q \otimes \sigma^{eq \to eq} \otimes F F^{q \to h}
\]

→ chiral-odd FF as a polarimeter of transv. quark polarization
Semi-Inclusive 2-Hadron Production
polarized 2-hadron cross section:

\[ \sigma_{UT} \sim \sin(\phi_{R \perp} + \phi_S) \sum e_q^2 h^q_1 H^{<}_1 \]

\[ H^{<}_1 = H^{<}_1(z, \zeta, M^2_{\pi\pi}) \]

\[ (\zeta \sim z_1/(z_1 + z_2)) \]
polarized 2-hadron cross section:

Unpolarized beam, Transversely pol. target

\[
\sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum e_q^2 h_1^{q} H_1^{<}
\]

\[
H_1^{<} = H_1^{<}(z, \zeta, M_{\pi\pi}^2)
\]

\[
(\zeta \sim z_1/(z_1 + z_2))
\]

- only relative momentum of hadron pair relevant

⇒ integration over transverse momentum of hadron pair simplifies factorization and \(Q^2\) evolution

- however, cross section becomes more complex (differential in 9 variables)
2-Hadron Fragmentation in Semi-Inclusive DIS

polarized 2-hadron cross section:
(\text{Unpolarized beam, Transversely pol. target})

$$\sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum e_q^2 h_1^q H_1^\perp$$

$$H_1^\perp = H_1^\perp(z, \zeta, M_{\pi\pi}^2)$$

$$\left(\zeta \sim \frac{z_1}{z_1 + z_2}\right)$$

difficult to measure directly \(\sigma_{UT} \equiv \sigma_{U\uparrow} - \sigma_{U\downarrow}\)

\(\Rightarrow\) measure cross section asymmetry \(A_{UT}\):

$$A_{UT} \equiv \frac{1}{\langle |S_T| \rangle} \frac{N_{2\pi}^{\uparrow}(\phi_{R\perp}, \phi_S, \theta) - N_{2\pi}^{\downarrow}(\phi_{R\perp}, \phi_S, \theta)}{N_{2\pi}^{\uparrow}(\phi_{R\perp}, \phi_S, \theta) + N_{2\pi}^{\downarrow}(\phi_{R\perp}, \phi_S, \theta)}$$

\(\uparrow \downarrow\) \ldots target spin states

\(N_{2\pi}\ldots\text{(norm.)} 2\pi\) yield

\(S_T\ldots\text{target polarization}\)
polarized 2-hadron cross section:

\( \sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum e_q^2 h^q_1 H^\lessgtr_1 \)

\( H^\lessgtr_1 = H^\lessgtr_1(z, \zeta, M^2_{\pi\pi}) \)

(\( \zeta \sim z_1/(z_1 + z_2) \))

difficult to measure directly \( \sigma_{UT} \equiv \sigma_{U\uparrow} - \sigma_{U\downarrow} \)

\( \Rightarrow \) measure cross section asymmetry \( A_{UT} \):

\[
A_{UT} \equiv \frac{1}{\langle |S_T| \rangle} \frac{N^\uparrow_{2\pi}(\phi_{R\perp}, \phi_S, \theta) - N^\downarrow_{2\pi}(\phi_{R\perp}, \phi_S, \theta)}{N^\uparrow_{2\pi}(\phi_{R\perp}, \phi_S, \theta) + N^\downarrow_{2\pi}(\phi_{R\perp}, \phi_S, \theta)}
\]

But: asymmetry involves unknown unpolarized \( \pi^0 \) cross section
Interference Fragmentation – Models

\[ A_{UT} \sim \sin(\phi_R + \phi_S) \sin \theta h_1 H_1^< \]

Expansion of \( H_1^< \) in Legendre moments:

\[ H_1^<(z, \cos \theta, M_{\pi \pi}^2) = H_1^{<,sp}(z, M_{\pi \pi}^2) + \cos \theta H_1^{<,pp}(z, M_{\pi \pi}^2) \]

describe interference between 2 pion pairs coming from different production channels.

about \( H_1^{<,sp} \):

Jaffe et al. [hep-ph/9709322]:

\[ H_1^{<,sp}(z, M_{\pi \pi}^2) = \sin\delta_0 \sin\delta_1 \sin(\delta_0 - \delta_1) H_1^{<,sp'}(z) \]

\( \delta_0 (\delta_1) \rightarrow \text{S(P)-wave phase shifts} \)

\[ -\mathcal{P}(M_{\pi \pi}^2) H_1^{<,sp'}(z) \]

\( \Rightarrow A_{UT} \) might depend strongly on \( M_{\pi \pi} \)
Interference Fragmentation – Models

\[ A_{UT} \sim \sin(\phi_R + \phi_S) \sin \theta h_1 H_1^L \]

Expansion of \( H_1^L \) in Legendre moments:

\[ H_1^L(z, \cos \theta, M_{\pi \pi}^2) = H_1^{L,sp}(z, M_{\pi \pi}^2) + \cos \theta H_1^{L,pp}(z, M_{\pi \pi}^2) \]

Radici et al. [hep-ph/0110252]:

- completely different model, not predicting a sign change of the asymmetry
Mass Dependence of $A_{UT}$

2002-04 Data

- 2-hadron (aka Interference) FF is not zero!
- Asymmetry grows with $M_{\pi\pi}$ below $\rho^0$ mass
- Positive asymmetries in all invariant mass bins
- Rules out predicted sign change at $\rho^0$ mass (Jaffe et al.)
- To extract transversity ($h_1$) need IFF from Belle (or BaBar etc.)
- Non-zero IFF shows feasibility of using it at, e.g., RHIC for transversity measurements
Semi-Inclusive 1-Hadron Production
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist

**Distribution Functions**

\[
\begin{align*}
    f_1 &= \uparrow \\
    g_1 &= \downarrow - \uparrow \\
    h_1 &= \uparrow - \downarrow \\
    f_{1T} &= \uparrow - \\
    h_{1T} &= \uparrow - \\
    h_{1L} &= \downarrow - \\
\end{align*}
\]

**Fragmentation Functions**

\[
\begin{align*}
    g_{1T} &= \downarrow - \\
    D_1 &= \uparrow \\
    G_1 &= \downarrow - \uparrow \\
    H_1 &= \uparrow - \\
    D_{1T} &= \uparrow - \\
    H_{1T} &= \uparrow - \\
    H_{1L} &= \downarrow - \\
\end{align*}
\]

Chiral-odd transversity \( h_1 \) must couple to chiral-odd FF
Chiral-odd transversity $h_1$ must couple to chiral-odd FF

⇒ $H_1$ is the only $k_T$-integrated chiral-odd FF ⇒ DSA

(Example: transverse-spin transfer in $\Lambda$-production)
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist Distribution Functions

\[ f_1 = \] 
\[ g_1 = \] 
\[ h_1 = \] 
\[ f_{1T} = \] 
\[ h_{1L} = \]

Fragmentation Functions

\[ D_1 = \] 
\[ G_1 = \] 
\[ H_1 = \] 
\[ D_{1T} = \] 
\[ H_{1L} = \] 
\[ H_{1T} = \]

Chiral-odd transversity \( h_1 \) must couple to chiral-odd FF can use \( k_T \)-unintegrated chiral-odd FF ⇒ T-odd Collins FF ⇒ leads to Single-Spin Asymmetrie (SSA)
Leading-Twist Distribution Functions

\[ f_1 = \quad \]
\[ g_1 = \quad \]
\[ h_1 = \quad \]

\[ f_{1T} = \quad \]
\[ h_{1T} = \quad \]
\[ h_{1L} = \quad \]

T-odd

SSAs require one and only one T-odd function

SSA & Unintegrated Distribution and Fragmentation Functions

\[ D_1 = \quad \]
\[ G_1 = \quad \]
\[ H_1 = \quad \]

\[ D_{1T} = \quad \]
\[ H_{1T} = \quad \]
\[ H_{1L} = \quad \]
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist Distribution Functions

\[ f_1 = \]
\[ g_1 = \]
\[ h_1 = \]
\[ f_{1T} = \]
\[ h_{1L} = \]

Feasibility:

SSAs require one and only one T-odd function.

\[ \Rightarrow \text{SSAs through Collins function} \]
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist

Distribution Functions

- $f_1 = \circ$
- $g_1 = \rightarrow - \rightarrow$
- $h_1 = \uparrow - \downarrow$
- $f_{1T} = \uparrow - \rightarrow$
- $h_{1L} = \rightarrow - \rightarrow$
- $h_{1L} = \rightarrow - \rightarrow$

Fragmentation Functions

- $D_1 = \circ$
- $G_1 = \rightarrow - \rightarrow$
- $G_{1T} = \uparrow - \rightarrow$
- $H_1 = \uparrow - \rightarrow$
- $D_{1T} = \uparrow - \rightarrow$
- $H_{1L} = \uparrow - \rightarrow$
- $H_{1L} = \uparrow - \rightarrow$

SSAs require one and only one T-odd function
$\Rightarrow$ SSAs through Collins function or Sivers function
(Boer-Mulders DF couples to $H_1$, but SSA requires polarization of final state!)
SIDIS Cross Section
(up to subleading order in $1/Q$)

\[ d\sigma = d\sigma^0_{UU} + \cos 2\phi \ d\sigma^1_{UU} + \frac{1}{Q} \cos \phi \ d\sigma^2_{UU} + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma^3_{LU} \]

\[ + S_L \left\{ \sin 2\phi \ d\sigma^4_{UL} + \frac{1}{Q} \sin \phi \ d\sigma^5_{UL} + \lambda_e \left[ d\sigma^6_{LL} + \frac{1}{Q} \cos \phi \ d\sigma^7_{LL} \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \ d\sigma^8_{UT} + \sin(\phi + \phi_S) \ d\sigma^9_{UT} + \sin(3\phi - \phi_S) \ d\sigma^{10}_{UT} \right. \]

\[ + \frac{1}{Q} \left( \sin(2\phi - \phi_S) \ d\sigma^{11}_{UT} + \sin \phi_S \ d\sigma^{12}_{UT} \right) \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \ d\sigma^{13}_{LT} + \frac{1}{Q} \left( \cos \phi_S \ d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \ d\sigma^{15}_{LT} \right) \right] \}

Bacchetta et al., JHEP 0702 (2007) 093
\[ d\sigma = d\sigma^0_{UU} + \cos 2\phi \ d\sigma^1_{UU} + \frac{1}{Q} \cos \phi \ d\sigma^2_{UU} + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma^3_{LU} \]

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\[ + S_T \left\{ \sin(\phi - \phi_S) \ d\sigma^8_{UT} + \sin(\phi + \phi_S) \ d\sigma^9_{UT} + \sin(3\phi - \phi_S) \ d\sigma^{10}_{UT} \right. \]

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\[ + \lambda_e \left[ \cos(\phi - \phi_S) \ d\sigma^{13}_{LT} + \frac{1}{Q} \left( \cos \phi_S \ d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \ d\sigma^{15}_{LT} \right) \right] \}

This talk: \( \sin \phi \ d\sigma^5_{UL} \) \( \ldots \) Subleading Twist

\( \sin(\phi - \phi_S) \ d\sigma^8_{UT} \) \( \ldots \) Sivers Effect

\( \sin(\phi + \phi_S) \ d\sigma^9_{UT} \) \( \ldots \) Collins Effect
SIDIS Cross Section
(up to subleading order in $1/Q$)

\[ d\sigma = d\sigma^0_{UU} + \cos 2\phi \, d\sigma^1_{UU} + \frac{1}{Q} \cos \phi \, d\sigma^2_{UU} + \lambda_e \frac{1}{Q} \sin \phi \, d\sigma^3_{LU} \]

\[ + S_L \left\{ \sin 2\phi \, d\sigma^4_{UL} + \frac{1}{Q} \sin \phi \, d\sigma^5_{UL} + \lambda_e \left[ d\sigma^6_{LL} + \frac{1}{Q} \cos \phi \, d\sigma^7_{LL} \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \, d\sigma^8_{UT} + \sin(\phi + \phi_S) \, d\sigma^9_{UT} + \sin(3\phi - \phi_S) \, d\sigma^{10}_{UT} \right. \]

\[ \left. + \frac{1}{Q} \left( \sin(2\phi - \phi_S) \, d\sigma^{11}_{UT} + \sin \phi_S \, d\sigma^{12}_{UT} \right) \right\} \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \, d\sigma^{13}_{LT} + \frac{1}{Q} \left( \cos \phi_S \, d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \, d\sigma^{15}_{LT} \right) \right] \}

Also Interesting:
\[ \sin \phi_S \, d\sigma^{12}_{UT}, \ \cos \phi_S \, d\sigma^{14}_{LT} \ldots \Rightarrow \text{Transversity, } g_2 \]
\[ \cos \phi \, d\sigma^2_{UU} \]
\[ \cos 2\phi \, d\sigma^1_{UU} \ldots \Rightarrow \text{Cahn Effect} \]
\[ \ldots \text{Boer-Mulders Effect} \]
Azimuthal Single-Spin Asymmetries

\[ A_{UT}(\phi, \phi_S) = \frac{1}{\langle |S_\perp| \rangle} \frac{N^\uparrow_{h}(\phi, \phi_S) - N^\downarrow_{h}(\phi, \phi_S)}{N^\uparrow_{h}(\phi, \phi_S) + N^\downarrow_{h}(\phi, \phi_S)} \]

\[ \sim \sin(\phi + \phi_S) \sum_q e_q^2 \mathcal{I} \left[ \frac{k_T \hat{P}_{h\perp}}{M_h} h_{1}^{q}(x, p_T^2) H_{1}^{\perp,q}(z, k_T^2) \right] \]

\[ + \sin(\phi - \phi_S) \sum_q e_q^2 \mathcal{I} \left[ \frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^{\perp,q}(x, p_T^2) D_{1}^{q}(z, k_T^2) \right] \]

\[ + \ldots \quad \mathcal{I}[\ldots]: \text{convolution integral over initial } (p_T) \text{ and final } (k_T) \text{ quark transverse momenta} \]

\[ \Rightarrow 2D \text{ Max. Likelihood fit of to get Collins and Sivers amplitudes} \]

\[ PDF(2\langle \sin(\phi \pm \phi_S) \rangle_{UT}, \ldots, \phi, \phi_S) = \frac{1}{2} \{1 + P_T(2\langle \sin(\phi \pm \phi_S) \rangle_{UT} \sin(\phi \pm \phi_S) + \ldots)\} \]
Side Remark:

Resolving the Convolution Integral

Weight with transverse hadron momentum $P_{h\perp}$ to resolve convolution:

\[
\tilde{A}_{UT}(\phi, \phi_S) = \frac{1}{\langle S_\perp \rangle} \left( \sum_{i=1}^{N^+} P_{h\perp,i} - \sum_{i=1}^{N^-} P_{h\perp,i} \right) \frac{N^+ + N^-}{N^+ + N^-}
\]

\[
\sim \sin(\phi + \phi_C) \cdot \sum_q e_q^2 h_{1T}^{q}(x) \approx H_{1T}^{q,1}(1) \cdot D_q(z)
\]

(1): $p_T^2$-$k_T^2$-moment of distribution / fragmentation function

\[
- \sin(\phi - \phi_S) \cdot \sum_q e_q^2 f_{1T}^{q,1}(x) \approx D_1^q(z)
\]

+ ...
Monte Carlo Test of the Extraction Method

- generate Collins and Sivers asymmetries (Gaussian Ansatz in $p_T^2$)
- analyze MC data like experimental data and extract amplitudes:

\[
A_{UT}^{\pi^+}(z) = A_{UT}^{\pi^-}(z) = A_{UT}^{\pi^0}(z) 
\]

Collins-Sivers cross contamination negligible

- insensitive to $\cos(2\phi)$ moments in unpolarized cross section
- insensitive to transverse target tracking corrections
Collins Amplitudes 2002-2005

- published† results confirmed with much higher statistical precision
- overall scale uncertainty of 8.1%
- positive for $\pi^+$ and negative for $\pi^-$ as maybe expected ($\delta u > 0$, $\delta d < 0$)
- unexpected large $\pi^-$ asymmetry
  $\Rightarrow$ role of disfavored Collins FF
  most likely: $H_{1}^{\perp, disf} \approx -H_{1}^{\perp, fav}$
- isospin symmetry among charged and neutral pions fulfilled

† [A. Airapetian et al, Phys. Rev. Lett. 94 (2005) 012002]
transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)
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$q\bar{q}$-pair with vacuum quantum numbers ($^3P_0$-state)
transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)

$L=1$

$q\bar{q}$-pair with vacuum quantum numbers ($^3P_0$-state)

outgoing pion/kaon deflected into page (positive Collins FF)
transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)

$q\bar{q}$-pair with vacuum quantum numbers ($^3P_0$-state)

outgoing pion/kaon deflected into page (positive Collins FF)

pion from next string break deflected out of page (negative Collins FF)
The Collins Effect

Artru Model vs. HERMES

\[ \phi_s = 0 \]

\[ \phi = \pi / 2 \]

\[ \sin(\phi + \phi_s) > 0 \]
The Collins Effect

Artru Model vs. HERMES

\[
\begin{align*}
\phi_S &= 0 \\
\phi &= \pi/2
\end{align*}
\]

\[\sin(\phi + \phi_S) > 0\]
\[ \begin{align*}
\phi_S &= 0 \\
\phi &= \frac{\pi}{2} 
\end{align*} \quad \sin(\phi + \phi_S) > 0 \]

\[ \begin{align*}
\phi_S &= \frac{\pi}{2} \\
\phi &= 0 
\end{align*} \quad \sin(\phi + \phi_S) > 0 \]
The Collins Effect

Artru Model vs. HERMES

\[ \phi_S = 0 \quad \phi = \pi/2 \quad \sin(\phi + \phi_S) > 0 \]

\[ \phi_S = \pi/2 \quad \phi = 0 \quad \sin(\phi + \phi_S) > 0 \]
Artru model and HERMES results in agreement!
(assuming $u$-quark transversity positive)
Artru model and HERMES results in agreement also for $\pi^-$! (e.g., assuming $h_1^u h_1^d < 0$ and using $H_1^u \rightarrow \pi^- H_1^d \rightarrow \pi^- < 0$)
HERMES PRELIMINARY 2002-2005
lepton beam asymmetry, Collins amplitudes
8.1% scale uncertainty

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Gunar Schnell, Universiteit Gent
Collins Amplitudes (2002-2005 data)

- none of the kaon amplitudes significantly non-zero
- however, \( K^+ \) amplitudes not different from \( \pi^+ \) amplitudes
- \( K^- \) amplitudes slightly positive, contrary to large negative \( \pi^- \) amplitudes
- \( K^- \) pure sea object
  \( \Rightarrow \) production dominated by u-quark scattering
Sivers Amplitudes
(2002-2005 data)

Published† results confirmed with much higher statistical precision

Overall scale uncertainty of 8.1%

\( \pi^+ \): positive; \( \pi^- \): consistent with zero

\( \Rightarrow \) first evidence for non-zero Sivers fct.: 
\[ f_{1T}^{⊥,u} < 0 \] (u-quark dominance)

\( \Rightarrow \) non-zero orbital angular momentum

Isospin symmetry for Sivers amplitudes fulfilled


† [A. Airapetian et al, Phys. Rev. Lett. 94 (2005) 012002]
approach by M. Burkardt:

spatial distortion of q-distribution
(obtained using anom. magn. moments & impact parameter dependent PDFs)
Chromodynamic Lensing
Understanding the Sivers Moments

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+ attractive QCD potential
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⇒ transverse asymmetries
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⇒ transverse asymmetries

$\mathbf{L}_\perp > 0$

u mostly over here

FSI kick

$\phi_S = \pi/2$

$\phi = \pi$

$\sin(\phi - \phi_S) > 0$

Gunar Schnell, Universiteit Gent
Sivers Amplitudes 2002-2005

- comparison with model calculation by Anselmino et al., based on:
  - Gaussian Ansatz for Sivers function
  - average transverse momenta from unpolarized \( \cos \phi \) amplitudes
  - non-zero Sivers function only for valence quarks
- excellent description of pion amplitudes from 2002-05 data
**Sivers Amplitudes 2002-2005**

- comparison with model calculation by Anselmino et al., based on:
  - Gaussian Ansatz for Sivers function
  - average transverse momenta from unpolarized \( \cos \phi \) amplitudes
  - non-zero Sivers function only for valence quarks
- excellent description of pion amplitudes from 2002-05 data
- fails to describe kaon amplitudes
Non-trivial role of sea quarks!
"Longitudinal" SSAs
Mixing of Azimuthal Moments

Experiment: Target Polarization w.r.t. Beam Direction (l)!

Theory: Polarization along virtual photon direction (q)

⇒ mixing of “experimental” and “theory” asymmetries via:


\[
\begin{pmatrix}
\langle \sin \phi \rangle_{UL} \\
\langle \sin(\phi - \phi_S) \rangle_{UT} \\
\langle \sin(\phi + \phi_S) \rangle_{UT}
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\
\frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\
\frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*}
\end{pmatrix}
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^q \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^q \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^q
\end{pmatrix}
\]

\(\cos \theta_{\gamma^*} \approx 1, \ \sin \theta_{\gamma^*} \text{ up to 15% at HERMES energies}\)
Mixing of Azimuthal Moments II

\[
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^l \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^l \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^l \\
\end{pmatrix} = 
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\end{pmatrix}
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^q \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^q \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^q \\
\end{pmatrix}
\]

solve for photon-axis moments:

\[
\langle \sin \phi \rangle_{UL}^q \simeq \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left( \langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right)
\]
Mixing of Azimuthal Moments II

\[
\begin{pmatrix}
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\end{pmatrix}
\]

solve for photon-axis moments:

\[
\langle \sin \phi \rangle^{q}_{UL} \approx \langle \sin \phi \rangle^{l}_{UL} + \sin \theta_{\gamma^*} \left( \langle \sin(\phi + \phi_S) \rangle^{l}_{UT} + \langle \sin(\phi - \phi_S) \rangle^{l}_{UT} \right)
\]

\[
\langle \sin(\phi \pm \phi_S) \rangle^{q}_{UT} \approx \langle \sin(\phi \pm \phi_S) \rangle^{l}_{UT}
\]

\[-\frac{1}{2} \sin \theta_{\gamma^*} \left( \langle \sin \phi \rangle^{l}_{UL} + \tan \theta_{\gamma^*} \langle \sin(\phi \mp \phi_S) \rangle^{l}_{UT} \right)\]

max. 0.4% absolute correction

max. 1% relative correction
What About Longitudinally Polarized Targets?

\[
\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left( \langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right)
\]

\[
\langle \sin \phi \rangle_{UL}^q \propto \frac{M}{Q} \mathcal{I} \left[ \hat{P}_{h \perp k_T} \left( \frac{M_h}{z M} g_1 G^\perp + x h_L H_1^\perp \right) \right.
\]

\[
+ \frac{\hat{P}_{h \perp p_T}}{M} \left( \frac{M_h}{z M} h_{1L}^\perp \tilde{H} - x f_L^\perp D_1 \right) \right]
\]


\[\Rightarrow\] they are all subleading-twist expressions!

\[
\langle \sin \phi \rangle_{UL}^l \quad \ldots \quad \text{Airapetian et al., Phys. Rev. Lett. 84 (2000) 4047}
\]

\[
\langle \sin(\phi \pm \phi_S) \rangle_{UT}^l \quad \ldots \quad \text{Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002}
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What About Longitudinally Polarized Targets?

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\]

- **twist-3 dominates measured asymmetries on longitudinally polarized targets!**
- **significantly positive for** \(\pi^+\)
- **consistent with zero for** \(\pi^-\)
- **twist-3 not necessarily small**

Conclusions
Summary

Most precise data on transversely polarized hydrogen presented
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- First evidence for non-zero "Interference" FF
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- $\sin \phi$ amplitudes on long. polar. target dominated by twist-3
Outlook

- polarized beam
polarized beam

⇒ $A_{LT}$ inclusive DIS: $g_2$
polarized beam

$\Rightarrow A_{LT}$ inclusive DIS: $g_2$

$\Rightarrow A_{LT}$ in $\pi$ production (measurement of twist-3 fragmentation function and transversity)
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- Analysis of whole data set for 2-hadron fragmentation

- Measurement of $pp$-interference in 2-hadron fragmentation
Extracting Quark Distributions

Purity Formalism

\[
A_{UT}^{\sin(\phi - \phi_S),h}(x) = C \cdot \frac{\sum_q e_q^2 f_{1T}^{q \perp}(1),q(x) \int dz \ D_{1}^{q,h}(z) A(x, z)}{\sum_{q'} e_{q'}^2 f_{1}^{q'}(x) \int dz \ D_{1}^{q',h}(z) A(x, z)}
\]

\[
= C \cdot \sum_q e_q^2 f_{1}^{q}(x) \frac{D_{1}^{q,h}(x)}{\sum_{q'} e_{q'}^2 f_{1}^{q'}(x) D_{1}^{q',h}(x)} \cdot \frac{f_{1T}^{q}(1),q}{f_{1}^{q}}(x)
\]

\[
= C \cdot \sum_q \mathcal{P}^{h}_{q}(x) \cdot \frac{f_{1T}^{q}(1),q}{f_{1}^{q}}(x)
\]

- **purities** are completely unpolarized objects → present Monte Carlo-tunes can be used
- probabilistic interpretation of purities possible
- “easy”: Sivers ← fragmentation function \((D_1)\) known
Extracting Quark Distributions

Purity Formalism

\[ A_{UT}^{\sin(\phi+\phi_S),h}(x) = C \cdot \frac{\sum_q e_q^2 h_1^q(x) \int dz \ H_1^{(1),q,h}(z) A(x,z)}{\sum_q' e_{q'}^2 f_{1'}^q(x) \int dz \ D_{1'}^{q',h}(z) A(x,z)} \]

\[ = C \cdot \sum_q e_q^2 f_1^q(x) \frac{\mathcal{H}_1^{(1),q,h}(x)}{\sum_q' e_{q'}^2 f_{1'}^q(x) D_{1'}^{q',h}(x)} \cdot \frac{h_1^q}{f_1^q}(x) \]

\[ = C \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{h_1^q}{f_1^q}(x) \]

- **purities** are completely **unpolarized** objects → present Monte Carlo-tunes can be used
- probabilistic interpretation of purities possible
- “easy”: Sivers ← fragmentation function \((D_1)\) known
- Collins: these purities still **depend on parametrization** of Collins FF function