Transversity and Transverse Momentum Dependent Distribution and Fragmentation Functions

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For the hermes Collaboration
HERMES at DESY

27.5GeV positron beam of HERA
atomic beam source
⇒ pure gas target
transversely pol. hydrogen
polarization \(\sim 75\%\)
other targets possible
How can one measure the chiral-odd transversity?
Need another chiral-odd object!
How can one measure the chiral-odd transversity?

Need another chiral-odd object!

⇒ Semi-Inclusive DIS

\[ \sigma_{ep \rightarrow ehX} = \sum_q h_1^q \otimes \sigma_{eq \rightarrow eq} \otimes FF^{q \rightarrow h} \]

\[ \downarrow \]

\[ \text{chiral-odd DF} \]

\[ \downarrow \]

\[ \text{chiral-odd FF} \]

\[ \text{CHIRAL EVEN} \]
How can one measure the chiral-odd transversity?

Need another chiral-odd object!

⇒ Semi-Inclusive DIS

\[
\sigma^{ep \rightarrow ehX} = \sum_{q} h_{1}^{q} \otimes \sigma^{eq \rightarrow eq} \otimes FF^{q \rightarrow h}
\]

\[\downarrow\]

chiral-odd DF

\[\downarrow\]

chiral-odd FF

→ chiral-odd FF as a polarimeter of transv. quark polarization
Semi-Inclusive 2-Hadron Production
polarized 2-hadron cross section:

\[ \sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum e_q^2 h_1^q H_1^\perp \]

\[ H_1^\perp = H_1^\perp(z, \zeta, M_{\pi\pi}^2) \]

\( (\zeta \sim z_1/(z_1 + z_2)) \)
polarized 2-hadron cross section:
(Unpolarized beam, Transversely pol. target)

\[ \sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum e_q^2 h_1^q H_1^< \]

\[ H_1^< = H_1^<(z, \zeta, M_{\pi\pi}^2) \]

\[ (\zeta \sim z_1 / (z_1 + z_2)) \]
polarized 2-hadron cross section:
(Unpolarized beam, Transversely pol. target)

\[
\sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum q^2 h_1^q H_1^{\perp}
\]

\[
H_1^{\perp} = H_1^{\perp}(z, \zeta, M_{\pi\pi}^2)
\]

(\(\zeta \sim z_1/(z_1 + z_2)\))

difficult to measure directly \(\sigma_{UT} \equiv \sigma_{U\uparrow} - \sigma_{U\downarrow}\)

\(\Rightarrow\) measure cross section asymmetry \(A_{UT}\):

\[
A_{UT} \equiv \frac{1}{\langle |S_T| \rangle} \frac{N_{2\pi}^{\uparrow}(\phi_{R\perp}, \phi_S, \theta) - N_{2\pi}^{\downarrow}(\phi_{R\perp}, \phi_S, \theta)}{N_{2\pi}^{\uparrow}(\phi_{R\perp}, \phi_S, \theta) + N_{2\pi}^{\downarrow}(\phi_{R\perp}, \phi_S, \theta)}
\]

\(\uparrow \downarrow\) \ldots target spin states

\(N_{2\pi} \ldots\) (norm.) \(2\pi\) yield

\(S_T\ldots\) target polarization
polarized 2-hadron cross section:

\[ \sigma_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sum q e_q^2 h_1^q H_1^\perp \]

\[ H_1^\perp = H_1^\perp(z, \zeta, M_{\pi\pi}^2) \]

\[ (\zeta \sim z_1/(z_1 + z_2)) \]

difficult to measure directly \( \sigma_{UT} \equiv \sigma_{U\uparrow} - \sigma_{U\downarrow} \)

\[ \Rightarrow \text{measure cross section asymmetry } A_{UT} : \]

\[ A_{UT} \equiv \frac{1}{\langle |S_T| \rangle} \frac{N_{2\pi}^{\uparrow}(\phi_{R\perp}, \phi_S, \theta) - N_{2\pi}^{\downarrow}(\phi_{R\perp}, \phi_S, \theta)}{N_{2\pi}^{\uparrow}(\phi_{R\perp}, \phi_S, \theta) + N_{2\pi}^{\downarrow}(\phi_{R\perp}, \phi_S, \theta)} \]

\[ \uparrow \downarrow \ldots \text{target spin states} \]

\[ N_{2\pi} \ldots \text{(norm.) } 2\pi \text{ yield} \]

\[ S_T \ldots \text{target polarization} \]

But: asymmetry involves unknown unpolarized \( 2\pi \) cross section
Interference Fragmentation – Models

\[ A_{UT} \sim \sin(\phi_{R_{\perp}} + \phi_S) \sin \theta h_1 H_1^< \]

Expansion of \( H_1^< \) in Legendre moments:

\[ H_1^< (z, \cos \theta, M_{\pi \pi}^2) = H_1^{<,sp} (z, M_{\pi \pi}^2) + \cos \theta H_1^{<,pp} (z, M_{\pi \pi}^2) \]

about \( H_1^{<,sp} \):

describe interference between 2 pion pairs coming from different production channels.

Jaffe et al. [hep-ph/9709322]:

\[ H_1^{<,sp} (z, M_{\pi \pi}^2) = \sin \delta_0 \sin \delta_1 \sin(\delta_0 - \delta_1) H_1^{<,sp'} (z) \]

\[ \delta_0 (\delta_1) \to \text{S(P)-wave phase shifts} \]

\[ - \mathcal{P}(M_{\pi \pi}^2) H_1^{<,sp'} (z) \]

\[ \Rightarrow A_{UT} \text{ might depend strongly on } M_{\pi \pi} \]
Interference Fragmentation – Models

\[ A_{UT} \sim \sin(\phi_{R\perp} + \phi_S) \sin \theta h_1 H_1^\triangleleft \]

Expansion of \( H_1^\triangleleft \) in Legendre moments:

\[ H_1^\triangleleft(z, \cos \theta, M_{\pi\pi}^2) = H_1^{\triangleleft, sp}(z, M_{\pi\pi}^2) + \cos \theta H_1^{\triangleleft, pp}(z, M_{\pi\pi}^2) \]

Radici et al. [hep-ph/0110252]:

- completely different model, not predicting a sign change of the asymmetry
2-hadron (aka Interference) FF is not zero!

- asymmetry grows with $M_{\pi\pi}$ below $\rho^0$ mass
- positive asymmetries in all invariant mass bins
- rules out predicted sign change at $\rho^0$ mass (Jaffe et al.)
- to extract transversity ($h_1$) need Interference FF from Belle (or BaBar etc.)
Mass Dependence of $A_{UT}$

- 2-hadron (Interference) FF is not zero, asymmetry grows with $M_{\pi\pi}$ below $\rho^0$ mass
- positive asymmetries in all invariant mass bins
- rules out predicted sign change at $\rho^0$ mass (Jaffe et al.)
- to extract transversity ($h_1$) need Interference FF from Belle (or BaBar etc.)

Details: talk by F. Giordano
Semi-Inclusive 1-Hadron Production
Chiral-odd transversity $h_1$ must couple to chiral-odd FF
Chiral-odd transversity $h_1$ must couple to chiral-odd FF

$\Rightarrow H_1$ is the only $k_T$-integrated chiral-odd FF $\Rightarrow$ DSA

(Example: transverse-spin transfer in $\Lambda$-production)
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist

**Distribution Functions**

\[ f_1 = \]

\[ g_1 = \]

\[ h_1 = \]

\[ f_{1T} = \]

\[ h_{1T} = \]

\[ h_{1L} = \]

**Fragmentation Functions**

\[ D_1 = \]

\[ G_1 = \]

\[ H_1 = \]

\[ D_{1T} = \]

\[ H_{1L} = \]

\[ H_{1T} = \]

Chiral-odd transversity \( h_1 \) must couple to chiral-odd FF

**can use** \( k_T \)-unintegrated chiral-odd FF \( \Rightarrow \) T-odd Collins FF

\( \Rightarrow \) leads to Single-Spin Asymmetrie (SSA)
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist

Distribution Functions

\[ f_1 = \]

\[ g_1 = \]

\[ h_1 = \]

\[ f_{1T} = \] \{ \]

\[ h_{1T} = \]

\[ h_{1L} = \]

Fragmentation Functions

\[ D_1 = \]

\[ G_1 = \]

\[ H_1 = \]

\[ D_{1T} = \] \{ \]

\[ H_{1T} = \]

\[ H_{1L} = \]

SSAs require one and only one T-odd function
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist Distribution Functions

\[ f_1 = \text{ } \]
\[ g_1 = \text{ } \]
\[ h_1 = \text{ } \]
\[ f_{1T} = \text{ } \]
\[ h_{1T} = \text{ } \]
\[ h^{\perp}_{1L} = \text{ } \]

Fragmentation Functions

\[ D_1 = \text{ } \]
\[ G_1 = \text{ } \]
\[ H_1 = \text{ } \]
\[ D_{1T} = \text{ } \]
\[ H_{1T} = \text{ } \]
\[ H^{\perp}_{1L} = \text{ } \]

SSAs require one and only one T-odd function
\[ \implies \text{SSAs through Collins function} \]
SSA & Unintegrated Distribution and Fragmentation Functions

Leading-Twist Distribution Functions

SSAs require one and only one T-odd function

⇒ SSAs through Collins function or Sivers function

(Boer-Mulders DF couples to $H_1$, but SSA requires polarization of final state!)
\[ d\sigma = d\sigma_{UU}^0 + \cos 2\phi \ d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi \ d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma_{LU}^3 \]

\[ + S_L \left\{ \sin 2\phi \ d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi \ d\sigma_{UL}^5 + \lambda_e \left[ d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi \ d\sigma_{LL}^7 \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \ d\sigma_{UT}^8 + \sin(\phi + \phi_S) \ d\sigma_{UT}^9 + \sin(3\phi - \phi_S) \ d\sigma_{UT}^{10} \right. \]

\[ + \left. \frac{1}{Q} \left( \sin(2\phi - \phi_S) \ d\sigma_{UT}^{11} + \sin \phi_S \ d\sigma_{UT}^{12} \right) \right\} \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \ d\sigma_{LT}^{13} + \frac{1}{Q} \left( \cos \phi_S \ d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) \ d\sigma_{LT}^{15} \right) \right] \}

\[ d\sigma = d\sigma_{UU}^0 + \cos 2\phi \, d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi \, d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi \, d\sigma_{LU}^3 \]

\[ + S_L \left\{ \sin 2\phi \, d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi \, d\sigma_{UL}^5 + \lambda_e \left[ d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi \, d\sigma_{LL}^7 \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \, d\sigma_{UT}^8 + \sin(\phi + \phi_S) \, d\sigma_{UT}^9 + \sin(3\phi - \phi_S) \, d\sigma_{UT}^{10} \right. \]

\[ \left. + \frac{1}{Q} \left( \sin(2\phi - \phi_S) \, d\sigma_{UT}^{11} + \sin \phi_S \, d\sigma_{UT}^{12} \right) \right\} \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \, d\sigma_{LT}^{13} + \frac{1}{Q} \left( \cos \phi_S \, d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) \, d\sigma_{LT}^{15} \right) \right] \]

**This talk:**  
\[ \sin \phi \, d\sigma_{LU}^3, \sin \phi \, d\sigma_{UL}^5 \ldots \text{Subleading Twist} \]
\[ \sin(\phi - \phi_S) \, d\sigma_{UT}^8 \ldots \text{Sivers Effect} \]
\[ \sin(\phi + \phi_S) \, d\sigma_{UT}^9 \ldots \text{Collins Effect} \]
\[ d\sigma = d\sigma_0 + \cos 2\phi \, d\sigma_1 + \frac{1}{Q} \cos \phi \, d\sigma_2 + \lambda_e \frac{1}{Q} \sin \phi \, d\sigma_3 \]

\[ + S_L \left\{ \sin 2\phi \, d\sigma_4 + \frac{1}{Q} \sin \phi \, d\sigma_5 + \lambda_e \left[ d\sigma_6 + \frac{1}{Q} \cos \phi \, d\sigma_7 \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \, d\sigma_8 + \sin(\phi + \phi_S) \, d\sigma_9 + \sin(3\phi - \phi_S) \, d\sigma_{10} \right. \]

\[ + \frac{1}{Q} \left( \sin(2\phi - \phi_S) \, d\sigma_{11} + \sin \phi_S \, d\sigma_{12} \right) \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \, d\sigma_{13} + \frac{1}{Q} \left( \cos \phi_S \, d\sigma_{14} + \cos(2\phi - \phi_S) \, d\sigma_{15} \right) \right] \}\]
Azimuthal Single-Spin Asymmetries

\[ A_{UT}(\phi, \phi_S) = \frac{1}{\langle |S_\perp| \rangle} \frac{N^\uparrow_h(\phi, \phi_S) - N^\downarrow_h(\phi, \phi_S)}{N^\uparrow_h(\phi, \phi_S) + N^\downarrow_h(\phi, \phi_S)} \]

\[ \sim \sin(\phi + \phi_S) \sum_q e_q^2 \mathcal{I} \left[ \frac{k_T \hat{P}_{h\perp}}{M_h} h^q_1(x, p^2_T) H^{\perp,q}_1(z, k^2_T) \right] \]

\[ + \sin(\phi - \phi_S) \sum_q e_q^2 \mathcal{I} \left[ \frac{p_T \hat{P}_{h\perp}}{M} f^{\perp,q}_{1T}(x, p^2_T) D^q_1(z, k^2_T) \right] \]

\[ + \cdots \]

\[ \mathcal{I}[\ldots]: \text{ convolution integral over initial (}p_T\text{) and final (}k_T\text{) quark transverse momenta} \]

\[ \Rightarrow \text{2D-fit of } A_{UT} \text{ to get Collins and Sivers asymmetries:} \]

\[ A_{UT}(\phi, \phi_S) = 2 \left\langle \sin(\phi - \phi_S) \right\rangle_{UT} \sin(\phi - \phi_S) + 2 \left\langle \sin(\phi + \phi_S) \right\rangle_{UT} \sin(\phi + \phi_S) \]
Resolving the Convolution Integral

**Weight** with transverse hadron momentum $P_{h\perp}$ to resolve convolution:

$$\tilde{A}_{UT}(\phi, \phi_S) = \frac{1}{\langle S_{\perp} \rangle} \sum_{i=1}^{N^+} P_{h\perp,i} - \sum_{i=1}^{N^-} P_{h\perp,i} \frac{N^+ + N^-}{N^+ + N^-}$$

$$\sim \sin(\phi + \phi_C) \cdot \sum_q e_q^2 h_1^q(x) \sim H_{1}^{\perp,1}(1),q(z) \quad (1): \ p_T^2/k_T^2\text{-moment of}
$$

distribution / fragmentation function

$$- \sin(\phi - \phi_S) \cdot \sum_q e_q^2 f_{1T}^{\perp,1}(1),q(x) \sim D_1^q(z)$$

$$+ \ldots$$

$\Rightarrow$ 2D-fit of $\tilde{A}_{UT}$ to get Collins and Sivers asymmetries:

$$\tilde{A}_{UT}(\phi, \phi_S) = M_\pi 2 \left\langle \frac{P_{h\perp}}{M_\pi} \sin(\phi + \phi_S) \right\rangle_{UT}(x, z) \sin(\phi + \phi_S)$$

$$+ M_p 2 \left\langle \frac{P_{h\perp}}{M_p} \sin(\phi - \phi_S) \right\rangle_{UT}(x, z) \sin(\phi - \phi_S)$$
Monte Carlo Test of the Extraction Method

- generate Collins and Sivers asymmetries (Gaussian Ansatz in $p_T^2$)
- analyze MC data like experimental data and extract asymmetries:

Collins-Sivers cross contamination negligible

insensitive to $\cos(2\phi)$ moments in unpolarized cross section

insensitive to transverse target tracking corrections
Collins Asymmetries 2002-2004

- Published† results confirmed with much higher statistical precision
- Overall scale uncertainty of 6.6%
- Positive for $\pi^+$ and negative for $\pi^-$ as maybe expected ($\delta u > 0$ $\delta d < 0$)
- Unexpected large $\pi^-$ asymmetry
  => role of disfavored Collins FF
  Most likely: $H_{1,\text{disf}}^\perp \approx -H_{1,\text{fav}}^\perp$
- Partially large contribution from decay of exclusively produced vector mesons

† [A. Airapetian et al, Phys. Rev. Lett. 94 (2005) 012002]
Collins Asymmetries 2002-2004

HERMES PRELIMINARY 2002-2004
lepton beam asymmetry amplitudes not corrected for acceptance and smearing

HERMES PRELIMINARY 2004
lepton beam asymmetry amplitudes 6.6% scale uncertainty

Details: Talk by U. Elschenbroich
transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)
transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)

$q\bar{q}$-pair with vacuum quantum numbers ($^3P_0$-state)
transverse spin of struck quark (polarization component in lepton scattering plane reversed by photoabsorption)

$q\bar{q}$-pair with vacuum quantum numbers ($^3P_0$-state)

outgoing pion deflected into page (positive Collins FF)
The Collins Effect

Artru Model vs. HERMES

\[
\begin{align*}
\phi_S &= 0 \\
\phi &= \frac{\pi}{2} \\
\sin(\phi + \phi_S) &> 0
\end{align*}
\]
The Collins Effect

Artru Model vs. HERMES

\[ \phi_S = 0 \]
\[ \phi = \frac{\pi}{2} \]
\[ \sin(\phi + \phi_S) > 0 \]

✓

HERMES PRELIMINARY 2002-2004

lepton beam asymmetry amplitudes not corrected for acceptance and smearing

6.6% scale uncertainty
The Collins Effect

Artru Model vs. HERMES

\[ \sin(\phi + \phi_s) > 0 \]

\[ \phi_s = 0 \]
\[ \phi = \pi/2 \]

✓

\[ \phi_s = \pi/2 \]
\[ \phi = 0 \]

\[ \sin(\phi + \phi_s) > 0 \]
The Collins Effect

Artru Model vs. HERMES

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The Collins Effect

Artru Model vs. HERMES

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\phi_S &= 0 \\
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\end{align*} \]
\[
\left\{ \sin(\phi + \phi_S) > 0
\right\}
\]

\[ \begin{align*}
\phi_S &= \pi/2 \\
\phi &= 0
\end{align*} \]
\[
\left\{ \sin(\phi + \phi_S) > 0
\right\}
\]

Artru model and HERMES results in agreement!
(assuming \( u \)-quark transversity positive)
Results on Sivers Amplitudes from 2002-2004 data

\[ 2 \left\langle \sin(\phi - \phi_S) \right\rangle_{UT} \propto - \sum_q c_q^2 T \left[ \frac{p_T \hat{P}_h}{M} f_{1T;u}^{\perp,q}(x, p_T^2) D_1^q(z, K_T^2) \right] \]

- \( \pi^+ \): positive; \( \pi^- \): consistent with zero
- \( \Rightarrow \) first evidence for non-zero Sivers fct.: \( f_{1T;u}^{\perp,u} < 0 \) (\( u \)-quark dominance)
- Exclusive \( \rho^0 \) asymmetry (2005 prel.):

\[ A_{UT}(\phi, \phi_S) \]

\( \Rightarrow \) small syst. error from vector mesons
Results on Sivers Amplitudes from 2002-2004 data

\[ 2 \left\langle \sin(\phi - \phi_S) \right\rangle_{UT} \propto - \sum_q e_q^2 T \left[ \frac{p_T P_{h\perp}}{M} f_{1T,q}^T(x, p^2_T) D^q_1(z, K^2_T) \right] \]

smaller VM contribution to kaon sample
Results on Sivers Amplitudes from 2002-2004 data

\[ 2 \left< \sin(\phi - \phi_S) \right>_{UT} \propto - \sum_q c_q^2 \mathcal{I} \left[ \frac{p_T \vec{P}_h}{M} f_{1T}^{1T,q}(x, p_T^2) D^q_1(z, K_T^2) \right] \]
Results on Sivers Amplitudes from 2002-2004 data

\[ 2 \langle \sin(\phi - \phi_S) \rangle_{UT} \propto - \sum_q c_q^2 T \left[ \frac{p_T \hat{P}_{h\perp}}{M} f_{1T}^q(x, p_T^2) D_1^q(z) \right] \]

Details: Talk by U. Elschenbroich

HERMES PRELIMINARY 2002-2004
not corrected for acceptance and smearing

6.6% scale uncertainty

VM fraction

Gunar Schnell, Universiteit Gent
Chromodynamic Lensing
Understanding the Sivers Moments

approach by M. Burkardt:

spatial distortion of $q$-distribution (obtained using anom. magn. moments & impact parameter dependent PDFs)
Chromodynamic Lensing

Understanding the Sivers Moments

approach by M. Burkardt:

spatial distortion of q-distribution
(obtained using anom. magn. moments & impact parameter dependent PDFs)

+ attractive QCD potential
  (gluon exchange)

⇒ transverse asymmetries

\[ \phi_S = \pi/2 \]
\[ \phi = \pi \]
\[ \sin(\phi - \phi_S) > 0 \]
Chromodynamic Lensing
Understanding the Sivers Moments

approach by M. Burkardt:

spatial distortion of $q$-distribution
(obtained using anom. magn. moments & impact parameter dependent PDFs)

+ attractive QCD potential
(gluon exchange)

$\Rightarrow$ transverse asymmetries

\[ L_z^{u} > 0 \]

$u$ mostly over here

FSI kick

$\phi_S = \pi/2$

$\phi = \pi$

\[ \sin(\phi - \phi_S) > 0 \]
Longitudinal SSAs
Mixing of Azimuthal Moments

Experiment: Target Polarization w.r.t. Beam Direction (l)!

Theory: Polarization along virtual photon direction (q)

⇒ mixing of “experimental” and “theory” asymmetries via:

\[
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^l \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^l \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^l
\end{pmatrix}
= \begin{pmatrix}
\cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\
\frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\
\frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*}
\end{pmatrix}
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^q \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^q \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^q
\end{pmatrix}
\]

\(\cos \theta_{\gamma^*} \simeq 1, \ \sin \theta_{\gamma^*} \text{ up to 15\% at HERMES energies}\)
Mixing of Azimuthal Moments II

\[
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^l \\
\langle \sin(\phi-\phi_S) \rangle_{UT}^l \\
\langle \sin(\phi+\phi_S) \rangle_{UT}^l
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\
\frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\
\frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*}
\end{pmatrix}
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^q \\
\langle \sin(\phi-\phi_S) \rangle_{UT}^q \\
\langle \sin(\phi+\phi_S) \rangle_{UT}^q
\end{pmatrix}
\]

solve for photon-axis moments:

\[
\langle \sin \phi \rangle_{UL}^q \simeq \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left( \langle \sin(\phi+\phi_S) \rangle_{UT}^l + \langle \sin(\phi-\phi_S) \rangle_{UT}^l \right)
\]
Mixing of Azimuthal Moments II

\[
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^l \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^l \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^l
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta_\gamma^* & -\sin \theta_\gamma^* & -\sin \theta_\gamma^* \\
\frac{1}{2} \sin \theta_\gamma^* & \cos \theta_\gamma^* & 0 \\
\frac{1}{2} \sin \theta_\gamma^* & 0 & \cos \theta_\gamma^*
\end{pmatrix}
\begin{pmatrix}
\langle \sin \phi \rangle_{UL}^q \\
\langle \sin(\phi - \phi_S) \rangle_{UT}^q \\
\langle \sin(\phi + \phi_S) \rangle_{UT}^q
\end{pmatrix}
\]

solve for photon-axis moments:

\[
\langle \sin \phi \rangle_{UL}^q \simeq \langle \sin \phi \rangle_{UL}^l + \sin \theta_\gamma^* \left( \langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right)
\]

\[
\langle \sin(\phi \pm \phi_S) \rangle_{UT}^q \simeq \langle \sin(\phi \pm \phi_S) \rangle_{UT}^l - \frac{1}{2} \sin \theta_\gamma^* \left( \langle \sin \phi \rangle_{UL}^l + \tan \theta_\gamma^* \langle \sin(\phi \mp \phi_S) \rangle_{UT}^l \right)
\]

max. 0.4% absolute correction  
max. 1% relative correction
What About Longitudinally Polarized Targets?

\[ \langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left( \langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right) \]

\[ \langle \sin \phi \rangle_{UL}^q \propto \frac{M}{Q} \mathcal{I} \left[ \frac{\hat{P}_{h \perp} k_T}{M_h} \left( \frac{M_h}{zM} g_1 G^\perp + x h_L H_1^\perp \right) \right. \]

\[ + \left. \frac{\hat{P}_{h \perp} p_T}{M} \left( \frac{M_h}{zM} h_{1L}^\perp \tilde{H} - x f_L^\perp D_1 \right) \right] \]


⇒ they are all subleading-twist expressions!

\[ \langle \sin \phi \rangle_{UL}^l \]

\[ \langle \sin(\phi \pm \phi_S) \rangle_{UT}^l \]


What About Longitudinally Polarized Targets?

\[
\langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left( \langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right)
\]

- twist-3 dominates measured asymmetries on longitudinally polarized targets!
- significantly positive for \( \pi^+ \)
- consistent with zero for \( \pi^- \)

The Other Longitudinal SSA

Longitudinally pol. beam & unpol. target $\Rightarrow$ subleading-twist

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[ x e(x) H_1^+(z) - \frac{M_h}{z M} h_1^+(x) E(z) \right]$$

$\Rightarrow$ for long time candidate to access $e(x)$

$h_1^+(x)$ contribution either assumed to be zero (T-odd!) or small (?)
longitudinally pol. beam & unpol. target \Rightarrow \text{subleading-twist}

\[
\left\langle \sin \phi \right\rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[ x e(x) H_1^\perp(z) - \frac{M_h}{z M} h_1^+(x) E(z) \right. \\
+ \left. \frac{M_h}{z M} f_1(x) G_1^\perp(z) - x g_1^+(x) D_1(z) \right. \\
+ \left. \frac{m_q}{M} h_1^+(x) D_1(z) - \frac{m_q}{M} f_1(x) H_1^+(z) \right]
\]

quark-mass suppressed \Rightarrow

The Other Longitudinal SSA

longitudinally pol. beam & unpol. target \Rightarrow \text{subleading-twist}

\[ \langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[ xe(x) H_1^\perp(z) - \frac{M_h}{zM} h_1^\perp(x) E(z) \right. \]
\[ \left. + \frac{M_h}{zM} f_1(x) G^\perp(z) - x g^\perp(x) D_1(z) \right] \]

- many terms contributing – difficult to separate
- maybe some terms small?

Extraction:

\[ 2 \langle \sin \phi \rangle_{LU} = \frac{\sum^+ \sin \phi_i |P^+_e| - \sum^- \sin \phi_i |P^-_e|}{\frac{1}{2} (N^+ + N^-)} \]

Vector Meson Contribution:
Max. possible contribution to systematic uncertainty estimated using PYTHIA MC (tuned for HERMES)
Comparisons with CLAS Results

- not so good agreement at high $z$

\[ \langle \sin \phi \rangle \propto f(y) \equiv \frac{y \sqrt{1 - y^2} + y^2}{2} \]
Comparisons with CLAS Results

- not so good agreement at high $z$
- have to correct for different $y$ range at CLAS and HERMES:

$$\langle \sin \phi \rangle_{LU} \propto f(y) \equiv \frac{2y\sqrt{(1-y)}}{1-y+y^2/2}$$

strong suppression at HERMES for high $z$ compared to CLAS

$\Rightarrow$ rescaling of asymmetries leads to good agreement
What can we learn from $A_{UL}$

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[ x e(x) H_1^\perp(z) - \frac{M_h}{z M} h_1^\perp(x) E(z) ight]$$

$$- x g_1^\perp(x) D_1(z) + \frac{M_h}{z M} f_1(x) G_1^\perp(z) \right]$$

any help from other observables to separate contributions?
What can we learn from $A_{UL}$

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[ xe(x) H_1^\perp(z) - \frac{M_h}{z M} h_1^\perp(x) E(z) 
- x g_1^\perp(x) D_1(z) + \frac{M_h}{z M} f_1(x) G_1^\perp(z) \right]$$

any help from other observables to separate contributions?

bullet $jet \, SIDIS \Rightarrow$ only $g_1^\perp$-term survives
\[ \langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} \mathcal{I} \left[ x e(x) H_{1\perp}^\perp(z) - \frac{M_h}{z M} h_{1\perp}^\perp(x) E(z) \right. \\
\left. - x g_{1\perp}^\perp(x) D_{1}(z) + \frac{M_h}{z M} f_1(x) G_{1\perp}^\perp(z) \right] \]

any help from other observables to separate contributions?

- **jet SIDIS** ⇒ **only** \( g_{1\perp}^\perp \)-term survives

- 2-hadron production:

\[ \sigma_{LU} \propto \sin \phi_{R\perp} \left[ x e(x) H_{1\perp}^\perp(z, \zeta, M^2_h) + \frac{1}{z} f_1(x) \tilde{G}^\perp(z, \zeta, M^2_h) \right] \]
Summary

- First evidence for non-zero Interference FF
- Non-vanishing Collins effect observed for $\pi^\pm$
- Most likely scenario: $H_{1,\text{disf}}^\perp \approx -H_{1,\text{fav}}^\perp$
- First evidence of T-odd Sivers distribution in DIS
- Significant positive Sivers asymmetries for positive pions and kaons $\Rightarrow L^u_z > 0$
- $\sin \phi$ amplitudes on long. polar. target dominated by twist-3
- Observation of significant non-zero beam-spin asymmetries
More data taking in 2005
⇒ doubled statistics
More data taking in 2005
⇒ doubled statistics
polarized beam
More data taking in 2005
⇒ doubled statistics

polarized beam

⇒ $A_{LT}$ in $\pi$ production (measurement of twist-3 fragmentation function and transversity)
More data taking in 2005
⇒ doubled statistics

polarized beam

⇒ $A_{LT}$ in $\pi$ production (measurement of twist-3 fragmentation function and transversity)

Extraction of $P_{h\perp}$-weighted asymmetries underway
More data taking in 2005
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polarized beam
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⇒ Model-independent interpretation of amplitudes possible
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polarized beam
⇒ $A_{LT}$ in $\pi$ production (measurement of twist-3 fragmentation function and transversity)

Extraction of $P_{h\perp}$-weighted asymmetries underway
⇒ Model-independent interpretation of amplitudes possible

Flavour decomposition of Sivers function
Extracting Quark Distributions

Purity Formalism

\[
A_{UT}^{\sin(\phi - \phi_S),h}(x) = C \cdot \frac{\sum_q e_q^2 f_{1T}^{q(1),q}(x) \int dz D_{1}^{q,h}(z) A(x,z)}{\sum_q e_q^2 f_1^{q}(x) \int dz D_{1}^{q',h}(z) A(x,z)}
\]

\[
= C \cdot \sum_q \frac{e_q^2 f_1^{q}(x) D_{1}^{q,h}(x)}{\sum_q e_q^2 f_1^{q}(x) D_{1}^{q',h}(x)} \cdot \frac{f_{1T}^{q(1),q}}{f_1^{q}}(x)
\]

\[
= C \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{f_{1T}^{1(1),q}}{f_1^{q}}(x)
\]

- purities are completely unpolarized objects \( \rightarrow \) present Monte Carlo-tunes can be used
- probabilistic interpretation of purities possible
- “easy”: Sivers \( \leftarrow \) fragmentation function \( (D_1) \) known
Extracting Quark Distributions

Purity Formalism

\[ A_{UT}^{\sin(\phi + \phi_S),h}(x) = C \cdot \frac{\sum_q e_q^2 h_q^1(x) \int dz \, H_1^{(1),q,h}(z) A(x,z)}{\sum_{q'} e_{q'}^2 f_{q'}^1(x) \int dz \, D_{1}^{q',h}(z) A(x,z)} \]

\[ = C \cdot \sum_q e_q^2 f_{q}^1(x) \frac{\mathcal{H}_1^{(1),q,h}(x)}{\mathcal{H}_1^{(1),q,h}(x)} \cdot \frac{h_q}{f_q^1}(x) \]

\[ = C \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{h_q}{f_q^1}(x) \]

- **purities** are completely unpolarized objects → present Monte Carlo-tunes can be used
- probabilistic interpretation of purities possible
- “easy”: Sivers ← fragmentation function \((D_1)\) known
- Collins: these purities still depend on parametrization of Collins FF function
Backup Slides
A Closer Look at Collins Asymmetries I

rewrite asymmetries in terms of favored and disfavored fragmentation:

- neglect strange quarks
- assume Gaussian $k_T$ dependence of Collins FF → can resolve convolution
- employ isospin symmetry among fragmentation functions, i.e.

\[
\begin{align*}
D_f & \equiv D(u \rightarrow \pi^+) \simeq D(d \rightarrow \pi^-) \simeq D(\bar{d} \rightarrow \pi^+) \simeq D(\bar{u} \rightarrow \pi^-) \\
D_d & \equiv D(d \rightarrow \pi^+) \simeq D(u \rightarrow \pi^-) \simeq D(\bar{u} \rightarrow \pi^+) \simeq D(\bar{d} \rightarrow \pi^-) \\
\frac{1}{2}(D_f + D_d) & \simeq D(u \rightarrow \pi^0) \simeq D(d \rightarrow \pi^0) \simeq D(\bar{d} \rightarrow \pi^0) \simeq D(\bar{u} \rightarrow \pi^0)
\end{align*}
\]

\[
\begin{align*}
\tilde{A}^{\pi^+}/\pi^-_{C}(x, z) & \propto \frac{(4\delta u + \delta \bar{d})H_{f/d} + (4\delta \bar{u} + \delta d)H_{d/f}}{(4u + \bar{d})D_{f/d} + (4\bar{u} + d)D_{d/f}} \\
\tilde{A}^{\pi^0}_{C}(x, z) & \propto \frac{[4(\delta u + \delta \bar{u}) + \delta d + \delta \bar{d}] (H_{f} + H_{d})}{[4(u + \bar{u}) + d + \bar{d}] (D_{f} + D_{d})}
\end{align*}
\]
A Closer Look at Collins Asymmetries II

express asymmetries in terms of flavor ratios:

\[
\begin{align*}
\tilde{A}_C^{\pi^+} &= \mathcal{K}(x, z) \frac{4 + \delta r \mathcal{H}}{4 + r \mathcal{D}} \\
\tilde{A}_C^{\pi^-} &= \mathcal{K}(x, z) \frac{4 \mathcal{H} + \delta r}{4 \mathcal{D} + r} \\
\tilde{A}_C^{\pi^0} &= \mathcal{K}(x, z) \frac{(4 + \delta r)(1 + \mathcal{H})}{(4 + r)(1 + \mathcal{D})}
\end{align*}
\]

Polarized Objects

<table>
<thead>
<tr>
<th>Unpolarized Objects</th>
<th>Mixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathcal{H} = \frac{H_d}{H_f})</td>
<td>(D = \frac{D_d}{D_f})</td>
</tr>
<tr>
<td>(\delta r = \frac{\delta d + 4 \delta \bar{u}}{\delta u + \frac{1}{4} \delta d})</td>
<td>(r = \frac{d + 4 \bar{u}}{u + \frac{1}{4} d})</td>
</tr>
</tbody>
</table>

\[3 \text{ constraints and 3 unknowns!}\]

e.g., CTEQ6,R1990 and Kretzer et al.
express asymmetries in terms of flavor ratios:

\[
\tilde{A}_C^{\pi^+} = K(x, z) \frac{4 + \delta r \mathcal{H}}{4 + r \mathcal{D}} \\
\tilde{A}_C^{\pi^-} = K(x, z) \frac{4 \mathcal{H} + \delta r}{4 \mathcal{D} + r} \\
\tilde{A}_C^{\pi^0} = K(x, z) \frac{(4 + \delta r)(1 + \mathcal{H})}{(4 + r)(1 + \mathcal{D})}
\]

The three asymmetries are not independent \((C(x, z) \equiv \frac{r(x) + 4 \mathcal{D}(z)}{r(x) \mathcal{D}(z) + 4})\):

\[
\tilde{A}_C^{\pi^+}(x, z) + C(x, z) \tilde{A}_C^{\pi^-}(x, z) - (1 + C(x, z)) \tilde{A}_C^{\pi^0}(x, z) = 0
\]

\(\Rightarrow 3\) constraints and \(3\) unknowns!

e.g., CTEQ6, R1990 and Kretzer et al.
express asymmetries in terms of flavor ratios:

\[
\begin{align*}
\tilde{A}_{\pi^+}^C &= K(x, z) \frac{4 + \delta r \mathcal{H}}{4 + r \mathcal{D}} \\
\tilde{A}_{\pi^-}^C &= K(x, z) \frac{4 \mathcal{H} + \delta r}{4 \mathcal{D} + r} \\
\tilde{A}_{\pi^0}^C &= K(x, z) \frac{(4 + \delta r)(1 + \mathcal{H})}{(4 + r)(1 + \mathcal{D})}
\end{align*}
\]

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<td>( \mathcal{K} )</td>
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<td>( \mathcal{K} = \frac{(\delta u + \frac{1}{4} \delta \bar{d}) z H_f}{(u + \frac{1}{4} \bar{d}) D_f} )</td>
</tr>
<tr>
<td>( \mathcal{D} )</td>
<td>( \frac{D_d}{D_f} )</td>
<td></td>
</tr>
<tr>
<td>( r )</td>
<td>( \frac{d + 4 \bar{u}}{u + \frac{1}{4} \bar{d}} )</td>
<td></td>
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</table>

E.g., CTEQ6, R1990 and Kretzer et al.

\( \Rightarrow 3 \text{ constraints and 3 unknowns!} \)
eliminate $\mathcal{K}$ and relate $\mathcal{H}$ to $\delta r$

$\Rightarrow$ scan solution space for $\mathcal{H}$ and $\delta r$ by sampling set of $(\tilde{A}^\pi_C^+, \tilde{A}^\pi_C^-, \tilde{A}^\pi_C^0)$

(around measured values according to statistical uncertainty)
eliminate $K$ and relate $H$ to $\delta r$

$\Rightarrow$ scan solution space for $H$ and $\delta r$ by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

(around measured values according to statistical uncertainty)
eliminate $K$ and relate $H$ to $\delta r$

$\Rightarrow$ scan solution space for $H$ and $\delta r$ by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

(around measured values according to statistical uncertainty)
eliminate $\mathcal{K}$ and relate $\mathcal{H}$ to $\delta r$

$\Rightarrow$ scan solution space for $\mathcal{H}$ and $\delta r$ by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

(around measured values according to statistical uncertainty)
eliminate $\kappa$ and relate $\mathcal{H}$ to $\delta r$

$\Rightarrow$ scan solution space for $\mathcal{H}$ and $\delta r$ by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

(around measured values according to statistical uncertainty)
eliminate $\mathcal{K}$ and relate $\mathcal{H}$ to $\delta r$

$\Rightarrow$ scan solution space for $\mathcal{H}$ and $\delta r$ by sampling set of $(\tilde{A}_C^{\pi^+}, \tilde{A}_C^{\pi^-}, \tilde{A}_C^{\pi^0})$

(around measured values according to statistical uncertainty)
probability distribution for $H_d/H_f$ vs. $\delta r$:
Limits on Transversity and Collins FF

$\delta r \approx \delta d/\delta u$ from $\chi_{QSM}$

look at slice of distribution in $\delta r$:

strong hint for $H_d/H_f$ negative