GMC_TRANS
- A Monte Carlo Generator for TMDs -

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Prelude:
The Role of Acceptance in Experiments
An unfortunate Lemma

“No particle-physics experiment has a perfect acceptance!”

- obvious for detectors with gaps/holes
- but also for “4π”, especially when looking at complicated final states
An unfortunate Lemma

“No particle-physics experiment has a perfect acceptance!”

HERMES azimuthal acceptance for 2-hadron production

“No particle-physics experiment has a perfect acceptance!”

maybe “4π” around beam axis, but not around virtual-photon axis because of lower limit on \( \theta \)

[W. Käfer, Transversity 2008, Ferrara]
“No particle-physics experiment has a perfect acceptance!”

momentum cuts strongly distort kinematic distributions even for “4π” acceptance
An unfortunate Lemma

“No particle-physics experiment has a perfect acceptance!”

- obvious for detectors with gaps/holes
- but also for “4π”, especially when looking at complicated final states
- How acceptance effects are handled is one of the essential questions in experiments!
Falsified Lemma I

“acceptance cancels in asymmetries”
Falsified Lemma I

“acceptance cancels in asymmetries”

\[ A_{UT}(\phi, \Omega) = \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \]

\[ \Omega = x, y, z, \ldots \]

\[ \epsilon : \text{detection efficiency} \]

\[ \int d\Omega \sigma_{UU}(\phi, \Omega) \equiv A_{UT}(\phi) \]
Falsified Lemma I

“acceptance cancels in asymmetries”

\[ A_{UT}(\phi, \Omega) = \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \]

\[ = \frac{\sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} \]

\[ \Omega = x, y, z, \ldots \]

\( \epsilon : \) detection efficiency

\[ \int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega) \]

\[ \approx \frac{\int d\Omega \sigma_{UT}(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} \]

\[ \Rightarrow A_{UT}(\phi) \]
Falsified Lemma I

- “acceptance cancels in asymmetries”

\[ A_{UT}(\phi, \Omega) = \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \]

\[ = \frac{\sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} \]

\[ \neq \frac{\int d\Omega \sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} \]

\[ \Omega = x, y, z, \ldots \]

\[ \epsilon : \text{detection efficiency} \]

\[ \int d\Omega \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \equiv A_{UT}(\phi) \]
Falsified Lemma I

“acceptance cancels in asymmetries”

\[
A_{UT}(\phi, \Omega) = \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)}
\]

\[
= \frac{\sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}
\]

\[
\neq \frac{\int d\Omega \sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)} \neq \frac{\int d\Omega \sigma_{UT}(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)} \equiv A_{UT}(\phi)
\]

\(\Omega = x, y, z, \ldots\)

\(\epsilon: \text{detection efficiency}\)
Falsified Lemma I

“acceptance cancels in asymmetries”

\[
A_{UT}(\phi, \Omega) = \frac{\sigma_{UT}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} = \frac{\sigma_{UT}(\phi, \Omega) \epsilon(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}
\equiv A_{UT}(\phi)
\]

\[
\Omega = x, y, z, \ldots
\]

\[
\epsilon : \text{detection efficiency}
\]

Acceptance does not cancel in general when integrating numerator and denominator over (large) ranges in kinematic variables!
... possible ways out

- for linear dependence on all kinematic variables of asymmetry, average asymmetry equal to asymmetry at average kinematics

- for all other cases: can one maybe use 1-D (projected) acceptance function, e.g. $\epsilon(\phi)$, to correct asymmetry $A_{UT}(\phi)$?
Falsified Lemma II

- use Monte Carlo (physics generator * detector model) to extract acceptance function
- “projected acceptance function is independent from cross-section model”
Falsified Lemma II

- use Monte Carlo (physics generator * detector model) to extract acceptance function

- “projected acceptance function is independent from cross-section model”

\[
\epsilon(\phi, \Omega) = \frac{\epsilon(\phi, \Omega) \sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \quad \Omega = x, y, z, \ldots
\]
**Falsified Lemma II**

- use Monte Carlo (physics generator * detector model) to extract acceptance function
- “projected acceptance function is independent from cross-section model”

\[
\epsilon(\phi, \Omega) = \frac{\epsilon(\phi, \Omega) \sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \quad \Omega = x, y, z, \ldots
\]

\[
\neq \frac{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)}
\]

\[
\int d\Omega \epsilon(\phi, \Omega) = \epsilon(\phi)
\]
Falsified Lemma II

- use Monte Carlo (physics generator * detector model) to extract acceptance function
- “projected acceptance function is independent from cross-section model”

\[
\epsilon(\phi, \Omega) = \frac{\epsilon(\phi, \Omega) \sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \\
\neq \frac{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)} \\
\neq \int d\Omega \epsilon(\phi, \Omega) \equiv \epsilon(\phi)
\]

\(\Omega = x, y, z, \ldots\)
Falsified Lemma II

- use Monte Carlo (physics generator * detector model) to extract acceptance function
- “projected acceptance function is independent from cross-section model”

\[
\epsilon(\phi, \Omega) = \frac{\epsilon(\phi, \Omega) \sigma_{UU}(\phi, \Omega)}{\sigma_{UU}(\phi, \Omega)} \quad \Omega = x, y, z, \ldots
\]

\[
\neq \frac{\int d\Omega \sigma_{UU}(\phi, \Omega) \epsilon(\phi, \Omega)}{\int d\Omega \sigma_{UU}(\phi, \Omega)}
\]

\[
\neq \int d\Omega \epsilon(\phi, \Omega) \equiv \epsilon(\phi)
\]

Cross-section model does not cancel in general when integrating numerator and denominator over (large) ranges in kinematic variables!
“Classique” Example: $\langle \cos \phi \rangle_{UU}$

1D correction
(input: MC without azimuthal modulation)

5D correction

[F. Giordano, Transversity 2008, Ferrara]
... one way out: multi-D unfolding

\[ n_{\text{corr}} = S_{\text{MC}}^{-1} [n_{\text{exp}} - BG_{\text{MC}}] \]

**true yield** (used to calculate azimuthal moments)

**experimental yield**

(inverted) multi-dimensional smearing matrix

(depends on detector and radiative effects only!)
... one way out: multi-D unfolding

true yield (used to calculate azimuthal moments)

(n_{corr} \rightarrow MC

(inverted) multi-dimensional smearing matrix

depends on detector and radiative effects only!

(experimental yield)
extracted Cahn amplitudes in good agreement with model amplitudes

apparent: need Cahn model for Monte Carlo simulation to test procedure

also needed to extrapolate into unmeasured region
1st Entrée: gmc_trans ingredients
Initial Goals

- physics generator for SIDIS pion production
- include transverse-momentum dependence, in particular simulate Collins and Sivers effects
- be fast
- allow comparison of input model and reconstructed amplitudes
- to be used with standard HERMES Monte Carlo
- be extendable (e.g., open for new models)
Basic workings

- use cross section that can (almost) be calculated analytically
- start from 1-hadron SIDIS expressions of Mulders & Tangerman (Nucl.Phys.B461:197-237,1996) and others
- use Gaussian Ansatz for all transverse-momentum dependencies of DFs and FFs
- unpolarized DFs (as well as helicity distribution) and FFs from fits/parametrizations (e.g., Kretzer FFs etc.)
- “polarized” DFs and FFs either related to unpolarized ones (e.g., saturation of Soffer bound for transversity) or some parametrizations used
SIDIS Cross Section incl. TMDs

\[ d\sigma_{UT} \equiv d\sigma^{\text{Collins}}_{UT} \cdot \sin(\phi + \phi_S) + d\sigma^{\text{Sivers}}_{UT} \cdot \sin(\phi - \phi_S) \]

\[ d\sigma^{\text{Collins}}_{UT}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} B(y) \sum_q e_q^2 \mathcal{I} \left[ \left( \frac{k_T \cdot \hat{P}_{h\perp}}{M_h} \right) \cdot h_1^q H_1^{\perp q} \right] \]

\[ d\sigma^{\text{Sivers}}_{UT}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \mathcal{I} \left[ \left( \frac{p_T \cdot \hat{P}_{h\perp}}{M_N} \right) \cdot f_{1T}^{\perp q} D_1^q \right] \]

\[ d\sigma_{UU}(x, y, z, \phi_S, P_{h\perp}) \equiv \frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \mathcal{I} \left[ f_{1T}^q D_1^q \right] \]

where

\[ \mathcal{I}[\mathcal{W} f D] \equiv \int d^2p_T d^2k_T \delta^{(2)} \left( p_T - \frac{P_{h\perp}}{z} - k_T \right) \left[ \mathcal{W} f(x, p_T) D(z, k_T) \right] \]
Gaussian Ansatz

- want to deconvolve convolution integral over transverse momenta

- easy Ansatz: Gaussian dependencies of DFs and FFs on intrinsic (quark) transverse momentum:

\[ \mathcal{I} [f_1(x, p_T^2) D_1(z, z^2 k_T^2)] = f_1(x) \cdot D_1(z) \cdot \frac{R^2}{\pi z^2} \cdot e^{-R^2 \frac{p_T^2}{z^2}} \]

with \[ f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \]

\[ \frac{1}{R^2} = \langle k_T^2 \rangle + \langle p_T^2 \rangle = \frac{\langle p_{h \perp}^2 \rangle}{z^2} \]

(similar: \( D_1(z, z^2 k_T^2) \))

Caution: different notations for intrinsic transverse momentum exist! (Here: Amsterdam notation)
**Positivity Constraints**

- DFs (FFs) have to fulfill various positivity constraints (resulting cross section has to be positive!)

- based on probability considerations can derive positivity limits for leading-twist functions:

- transversity: e.g., Soffer bound

- Sivers and Collins functions: e.g., loose bounds:

\[
\frac{|p_T|}{2M_N} f_{1T}^\perp (x, p_T^2) \equiv f_{1T}^{1/2} (x, p_T^2) \leq \frac{1}{2} f_1 (x, p_T^2)
\]

\[
\frac{|k_T|}{2M_h} H_1^\perp (z, z^2 k_T^2) \equiv H_1^{1/2} (z, z^2 k_T^2) \leq \frac{1}{2} D_1 (z, z^2 k_T^2)
\]
Positivity and the Gaussian Ansatz

\[
\frac{|p_T|}{2M_N} f_{1T}^\perp (x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2)
\]

with

\[
f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}
\]

\[
f_{1T}^\perp (x, p_T^2) = f_{1T}^\perp (x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}
\]

\[
|p_T| f_{1T}^\perp (x) \leq M_N f_1(x)
\]
Positivity and the Gaussian Ansatz

\[ \frac{|p_T|}{2M_N} f_{1T}^\perp(x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2) \]

with

\[ f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \]

\[ f_{1T}^\perp(x, p_T^2) = f_{1T}^\perp(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \]

\[ |p_T| f_{1T}^\perp(x) \leq M_N f_1(x) \]

No (useful) solution for non-zero Sivers fctn!
Modify Gaussian Width

\[ f_{1T}^\perp(x, p_T^2) = f_{1T}^\perp(x) \frac{1}{(1 - C)\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{(1 - C)\langle p_T^2 \rangle}} \]

→ positivity limit:

\[ f_{1T}^\perp(x) \frac{|p_T|}{2M_N} \frac{1}{\pi (1 - C) \langle p_T^2 \rangle} e^{-\frac{p_T^2}{(1 - C)\langle p_T^2 \rangle}} \leq \frac{1}{2} f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}} \]

\[ \frac{|p_T|}{1 - C} e^{-\frac{C}{1 - C} \frac{p_T^2}{\langle p_T^2 \rangle}} \leq M_N \frac{f_1(x)}{f_{1T}^\perp(x)} \]
Reevaluation of positivity constraint

\[
\frac{|p_T|}{1 - C} \exp\left(-\frac{C}{1 - C} \frac{p_T^2}{\langle p_T^2 \rangle}\right) \leq M_N \frac{f_1(x)}{f_{1T}(x)}
\]

Minimum at \( p_T = \sqrt{\frac{\langle p_T^2 \rangle}{2C}} \)

thus \( \frac{f_{1T}(x)}{f_1(x)} \leq M_N \sqrt{\frac{2eC(1-C)}{\langle p_T^2 \rangle}} \)

or \( \frac{f_{1T}^{(1/2)}(x)}{f_1(x)} \leq \frac{1}{2} \sqrt{\frac{e\pi C}{2}} (1 - C) \leq 0.4 \)
SIDIS Cross Section incl. TMDs

\[ \sum_{q} \frac{e_q^2}{4\pi} \frac{\alpha^2}{(M_{Eyxz})^2} \left[ X_{UU} + |S_T|X_{SIV} \sin(\phi_h - \phi_s) + |S_T|X_{COL} \sin(\phi_h + \phi_s) \right] \]

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

\[ X_{UU} = R^2 e^{-R^2 \frac{P_{h\perp}}{z^2}} \left( 1 - y + \frac{y^2}{2} \right) f_1(x) \cdot D_1(z) \]

\[ X_{COL} = + \frac{|P_{h\perp}|}{M_{\pi z}} \left( 1 - C \right) \langle k_T^2 \rangle \exp \left[ - \frac{P_{h\perp}^2 / z^2}{\langle p_T^2 \rangle + (1 - C) \langle k_T^2 \rangle} \right] \times \left( 1 - y \right) \cdot h_1(x) \cdot H_1^\perp(z) \]

\[ X_{SIV} = - \frac{|P_{h\perp}|}{M_{p z}} \left( 1 - C' \right) \langle p_T^2 \rangle \exp \left[ - \frac{P_{h\perp}^2 / z^2}{\langle k_T^2 \rangle + (1 - C') \langle p_T^2 \rangle} \right] \times \left( 1 - y + \frac{y^2}{2} \right) f_{1T}^\perp(x) \cdot D_1(z) \]
Sivers (azimuthal) Moments

use cross section expressions to evaluate azimuthal moments:

\[-\langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{\sqrt{(1 - C)\langle p_T^2 \rangle}}{\sqrt{(1 - C)\langle p_T^2 \rangle + \langle k_T^2 \rangle}} \]

\[-\langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{M_N \sqrt{\pi}}{2\sqrt{(1 - C)\langle p_T^2 \rangle + \langle k_T^2 \rangle}} \]

\[-\langle \frac{|P_{h\perp}|}{z M_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{2\sqrt{(1 - C)\langle p_T^2 \rangle}}{M_N \sqrt{\pi}} \]

model-dependence on transverse momenta “swallowed” by \( p_T^2 \)- moment of Sivers fct.: \( f_1^{_{\perp}(1)} \)

\[-\langle \frac{|P_{h\perp}|}{z M_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{_{\perp}(1/2)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)} \]

\[-\langle \frac{|P_{h\perp}|}{z M_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{_{\perp}(1)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)} \]
Sivers (azimuthal) Moments

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- \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{\sqrt{(1 - C) \langle p_T^2 \rangle}}{\sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} 
\]

\[
A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x) D_1(z)
\]

\[
- \langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{M_N \sqrt{\pi}}{2 \sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} 
\]

\[
A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1)}(x) D_1(z)
\]

\[
- \left\langle \frac{|P_{h\perp}|}{zM_N} \sin(\phi - \phi_s) \right\rangle_{UT} = \frac{2 \sqrt{(1 - C) \langle p_T^2 \rangle}}{M_N \sqrt{\pi}} 
\]

\[
A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x) D_1(z)
\]

\[
- \left\langle \frac{|P_{h\perp}|}{zM_N} \sin(\phi - \phi_s) \right\rangle_{UT} = \frac{M_N \sqrt{\pi}}{2 \sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} 
\]

\[
A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1)}(x) D_1(z)
\]

\[
- \left\langle \frac{|P_{h\perp}|}{zM_N} \sin(\phi - \phi_s) \right\rangle_{UT} = \frac{\sqrt{(1 - C) \langle p_T^2 \rangle}}{\sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} 
\]

\[
A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x) D_1(z)
\]

\[
- \left\langle \frac{|P_{h\perp}|}{zM_N} \sin(\phi - \phi_s) \right\rangle_{UT} = \frac{M_N \sqrt{\pi}}{2 \sqrt{(1 - C) \langle p_T^2 \rangle + \langle k_T^2 \rangle}} 
\]

\[
A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1)}(x) D_1(z)
\]

(similar for Collins moments)
2nd Entrée: Selected Results
Tuning the Gaussians in gmc_trans

- constant Gaussian widths, i.e., no dependence on x or z:
  \[ \langle p_T \rangle = 0.44 \]
  \[ \langle K_T \rangle = 0.44 \]
- tune to data integrated over whole kinematic range
Tuning the Gaussians in gmc_trans

\[ \langle p_T \rangle = 0.38 \]
\[ \langle K_T \rangle = 0.38 \]
\[
\langle p_T^2 \rangle = \langle K_T^2 \rangle = 0.18 \text{GeV}^2 \quad (\langle |\vec{p}_T| \rangle = \langle |\vec{K}_T| \rangle = 0.38 \text{GeV})
\]

where: \[
\langle K_T^2 \rangle = z^2 \langle k_T^2 \rangle
\]
Tuning the Gaussians in gmc_trans

in general: \[ \langle P_{h\perp}^2(x, z) \rangle = z^2 \langle p_T^2(x) \rangle + \langle K_T^2(z) \rangle \]

so far: \[ \langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle \]

constant!
Tuning the Gaussians in gmc_trans

so far: \[ \langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2 \rangle \]

\[ \langle p_T \rangle = 0.38 \]
\[ \langle K_T \rangle = 0.38 \]
\[ \langle p_T^2 \rangle \simeq 0.185 \]
\[ \langle K_T^2 \rangle \simeq 0.185 \]
Tuning the Gaussians in gmc_trans now:

\[ \langle P^2_{h\perp}(z) \rangle = z^2 \langle p^2_T \rangle + \langle K^2_T(z) \rangle \]

now \( z \)-dependent!

tuned to HERMES data in acceptance

"Hashi set"
Tuning the Gaussians in gmc_trans

\[ \langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2(z) \rangle \]

z-dependent!
Some Simple Models for Transversity & Friends

\[
\begin{align*}
\delta u(x) &= 0.7 \cdot \Delta u(x) & f_{1T}^{u}(x) &= -0.3 \cdot u(x) \\
\delta d(x) &= 0.7 \cdot \Delta d(x) & f_{1T}^{d}(x) &= 0.9 \cdot d(x) \\
\delta q(x) &= 0.3 \cdot \Delta q(x) & f_{1T}^{q}(x) &= 0.0 & q = \bar{u}, \bar{d}, s, \bar{s} \\
H_{1,\text{fav}}^{(1)}(z) &= 0.65 \cdot D_{1,\text{fav}}(z) \\
H_{1,\text{dis}}^{(1)}(z) &= -1.30 \cdot D_{1,\text{dis}}(z)
\end{align*}
\]

GRSV for PDFs and Kretzer FF for \(D_{1}\)
\begin{align*}
\delta u(x) &= 0.7 \cdot \Delta u(x) \\
\delta d(x) &= 0.7 \cdot \Delta d(x) \\
\delta q(x) &= 0.3 \cdot \Delta q(x)
\end{align*}

\begin{align*}
f_{1T}^{u}(x) &= -0.3 \cdot u(x) \\
f_{1T}^{d}(x) &= 0.9 \cdot d(x) \\
f_{1T}^{q}(x) &= 0.0
\end{align*}

\begin{align*}
H_{1,\text{fav}}^{(1)}(z) &= 0.65 \cdot D_{1,\text{fav}}(z) \\
H_{1,\text{dis}}^{(1)}(z) &= -1.30 \cdot D_{1,\text{dis}}(z)
\end{align*}

\[C_S = C_C = 0.25\]
Comparison for Weighted Moments

Not so good news for weighted moments

Graphs showing comparisons for weighted moments with circles highlighting specific data points.
GMC vs. Data Amplitudes

\[ 2 \sin(\phi_{\perp} - \phi_{\parallel}) \]

\[ \delta u(x) = 0.7 \cdot \Delta u(x) \quad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) \]
\[ \delta d(x) = 0.7 \cdot \Delta d(x) \quad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) \]
\[ \delta q(x) = 0.3 \cdot \Delta q(x) \quad f_{1T}^{\perp q}(x) = 0.0 \quad q = \bar{u}, d, s, \bar{s} \]

\[ H_{1,\text{fav}}^{(1)}(z) = 0.65 \cdot D_{1,\text{fav}}(z) \]
\[ H_{1,\text{dis}}^{(1)}(z) = -1.30 \cdot D_{1,\text{dis}}(z) \]

\[ C_S = C_G = 0.25 \]
**GMC vs. Data Amplitudes**

\[ \delta u(x) = 0.7 \cdot \Delta u(x) \]
\[ f_{1T}^{u}(x) = -0.6 \cdot u(x) \]
\[ H_{1,\text{fav}}^{(1)}(z) = 0.65 \cdot D_{1,\text{fav}}(z) \]
\[ \delta d(x) = 0.7 \cdot \Delta d(x) \]
\[ f_{1T}^{d}(x) = 1.05 \cdot d(x) \]
\[ H_{1,\text{dis}}^{(1)}(z) = -1.30 \cdot D_{1,\text{dis}}(z) \]
\[ \delta q(x) = 0.3 \cdot \Delta q(x) \]
\[ f_{1T}^{q}(x) = 0.3 \cdot q(x) \]

\( q = \bar{u}, d, s, \bar{s} \)

"hashi" set III for transverse momentum widths

\[ C_S = C_C = 0.25 \]

Transversity 2008, Beijing
Positivity Limits Revisited

- least stringent positivity constraints only involve considered ‘polarized’ function and the corresponding unpolarized function, e.g., Sivers vs. $f_1$
- more stringent relations arise from full density density matrix (e.g., Soffer relation for $h_1$ vs. $g_1$ and $f_1$):


\[
\begin{pmatrix}
  f_1 + g_1L \\
  \frac{|p_T|}{M} e^{-i\phi} (g_{1T} - if_{1T}^\perp) \\
  \frac{|p_T|}{M} e^{i\phi} (h_{1L}^\perp - i h_{1T}^\perp) \\
  2h_1
\end{pmatrix}
\begin{pmatrix}
  |p_T| e^{i\phi} (g_{1T} + if_{1T}^\perp) \\
  f_1 - g_1L \\
  \frac{|p_T|^2}{M^2} e^{2i\phi} h_{1T}^\perp \\
  -\frac{|p_T|^2}{M^2} e^{-i\phi} (h_{1L}^\perp + ih_{1T}^\perp)
\end{pmatrix}
\begin{pmatrix}
  \frac{|p_T|}{M} e^{-i\phi} (h_{1L}^\perp + ih_{1T}^\perp) \\
  f_1 - g_1L \\
  \frac{|p_T|^2}{M^2} e^{-2i\phi} h_{1T}^\perp \\
  -\frac{|p_T|^2}{M^2} e^{-i\phi} (h_{1L}^\perp - ih_{1T}^\perp)
\end{pmatrix}
\begin{pmatrix}
  2h_1 \\
  f_1 - g_1L \\
  -\frac{|p_T|^2}{M^2} e^{i\phi} (g_{1T} - if_{1T}^\perp) \\
  f_1 + g_1L
\end{pmatrix}
\]

has to be positive definite!
Positivity Limits Revisited

- least stringent positivity constraints only involve considered ‘polarized’ function and the corresponding unpolarized function, e.g., Sivers vs. $f_1$

- more stringent relations arise from full density density matrix (e.g., Soffer relation for $h_1$ vs. $g_1$ and $f_1$):


\[
\begin{pmatrix}
  f_1 + g_L \\
  \frac{|p_T|}{M} e^{i\phi} (g_T^+ + i f_{1T}^+) \\
  \frac{|p_T|}{M} e^{-i\phi} (g_T^- - i f_{1T}^-) \\
  2 h_1
\end{pmatrix}
\begin{pmatrix}
  f_1 - g_L \\
  \frac{|p_T|}{M} e^{i\phi} (g_T^- - i f_{1T}^-) \\
  \frac{|p_T|^2}{M^2} e^{2i\phi} (g_T^+ + if_{1T}^+) \\
  \frac{|p_T|^2}{M^2} e^{-2i\phi} (g_T^- - if_{1T}^-)
\end{pmatrix}
\begin{pmatrix}
  f_1 - g_L \\
  \frac{|p_T|}{M} e^{i\phi} (g_T^- - i f_{1T}^-) \\
  \frac{|p_T|^2}{M^2} e^{2i\phi} (g_T^+ + if_{1T}^+) \\
  \frac{|p_T|^2}{M^2} e^{-2i\phi} (g_T^- - if_{1T}^-)
\end{pmatrix}
\begin{pmatrix}
  2 h_1 \\
  - \frac{|p_T|}{M} e^{-i\phi} (h_L^+ + i h_{1T}^+) \\
  - \frac{|p_T|^2}{M^2} e^{-2i\phi} (h_L^- - i h_{1T}^-) \\
  f_1 + g_L
\end{pmatrix}
\]

has to be positive definite!
Positivity Limits Revisited

- least stringent positivity constraints only involve considered ‘polarized’ function and the corresponding unpolarized function, e.g., Sivers vs. \( f_1 \)

- more stringent relations arise from full helicity density matrix, e.g., Soffer relation for transversity vs. \( g_1 \) and \( f_1 \)


- reanalysis yields a more complex positivity limit for Sivers:

\[
\frac{p_T^2}{M^2} \left( f_{1T}^\perp (x, p_T^2) \right)^2 \leq f_1(x, p_T^2) \left( f_1(x, p_T^2) - 2|h_1(x, p_T^2)| \right)
\]

(required setting all other DF to zero)
“Critique de Ferrara”

“positivity limit has to involve either only $f_1$ or (almost) all PDFs”

implemented positivity limit in gmc_trans (self-consistently) involves only all in gmc_trans non-zero PDFs: $f_1$, $h_1$, Sivers (and if wanted BM)

however, we know $g_1$ is not zero, thus at least $g_1$ has to be considered as well for gmc_trans to be realistic:

$$\frac{p_T^2}{M^2} \left( f_{1T}^2(x, p_T^2) \right)^2 \leq \left\{ \left( f_1(x, p_T^2) \right)^2 - (g_1(x, p_T^2))^2 \right\} \left\{ 1 - \frac{2 |h_1(x, p_T^2)|}{f_1(x, p_T^2) + g_1(x, p_T^2)} \right\}$$

which positivity constraint is stronger and what about other PDFs?
Positivity for $g_1 = 0$ vs. $g_1 \neq 0$

$$
\Delta(\text{P.L.}) = \left(g_1(x, p_T^2)\right)^2 - 2 g_1(x, p_T^2) |h_1(x, p_T^2)|
$$

- for $g_1 < 0$ (e.g., down quarks) $g_1=0$ limit is \textless{} strict
- for $g_1 > 0$ (e.g., u-quarks) it depends on size of $h_1$
  ➞ so far in gmc\_trans productions $g_1=0$ limit was always less strict
- nevertheless, now implemented P.L. check that involves $g_1$ (and also BM function)
  ➥ should check P.L. involving all functions, but too little known about other TMDs
“Problem” with Sea Quarks

- Affected mainly (and strongly) strange-quark positivity limit

Check positivity limit for Transversity:

------
iquark= -3: lhs= 24.5845945 > rhs= 0.5
iquark= -2: lhs= 0.48761702 < rhs= 0.5
iquark= -1: lhs= 0.08159421 < rhs= 0.5
iquark=  1: lhs= 0.35142772 < rhs= 0.5
iquark=  2: lhs= 0.25735636 < rhs= 0.5

------
iquark=  3: lhs= 24.5845945 > rhs= 0.5
The Strange(!) Sea

- $f_1(x) + g_1(x)$ for strange quarks
- $\text{LST}(15) = 118$; 'standard' scenario, leading order GRSV
- $Q^2=1$
- Soffer bound: $|h_1(x)| < 0.5\{f_1(x)+g_1(x)\}$ can’t be fulfilled for nonzero $h_1$
<table>
<thead>
<tr>
<th>Quark</th>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0.38756798</td>
<td>0.38756827</td>
</tr>
<tr>
<td>-2</td>
<td>0.01550272</td>
<td>0.38756827</td>
</tr>
<tr>
<td>-1</td>
<td>0.15502719</td>
<td>0.38756827</td>
</tr>
</tbody>
</table>

Sivers & Transversity Fits by Anselmino et al.

- Sivers function rather close to positivity limit for anti-s
- Sivers function for d quarks 20% too large
Sivers & Transversity Fits by Anselmino et al.

$g_1 = 0$

Check positivity limit for Transversity:

- iquark = -3: lhs = 0. < rhs = 0.5
- iquark = -2: lhs = 0. < rhs = 0.5
- iquark = -1: lhs = 0. < rhs = 0.5
- iquark = 1: lhs = 0.221290307 < rhs = 0.5
- iquark = 2: lhs = 0.356384362 < rhs = 0.5
- iquark = 3: lhs = 0. < rhs = 0.5

Check positivity limit for T-odd DFs:

- iquark = -3: lhs = 0.38756798 < rhs = 0.38756827
- iquark = -2: lhs = 0.01550272 < rhs = 0.38756827
- iquark = -1: lhs = 0.15502719 < rhs = 0.38756827
- iquark = 1: lhs = 0.58568429 > rhs = 0.38756827
- iquark = 2: lhs = 0.21341756 < rhs = 0.38756827
- iquark = 3: lhs = 0.09301631 < rhs = 0.38756827

Non-Trivial Role of other TMDs!

- Sivers function rather close to positivity limit for anti-s
- Sivers function for d quarks 20% too large
(finally) Dessert:
the leaf of mint on the cake
certainly would like to model all TMDs, e.g., Boer-Mulders function, to get full cross section

even go to subleading-twist, e.g., Cahn effect

first attempts to implement those have been made

leading twist -- “straight forward” (just a few more convolution integrals)

subleading twist -- “hmmmm...”

biggest problem there: positivity limits don’t exist on DF and FF level
Current ToDo and Done List

- finish leading-twist implementation
- implement newest results from fits and model calculations on transversity, Sivers & Collins, ...
- add radiative corrections (e.g., RADGEN)
- make it portable to other experiments

(since Ferrara meeting:)
- ✓ Charged kaons and protons
- ✓ DSS FFs and published fits by Anselmino et al.
- ✓ neutron target
  - comparison of HERMES and COMPASS data possible (but not yet done)
Epilogue
Acceptance plays crucial part in analysis of multi-particle final states

Acceptance studies and/or corrections (e.g., unfolding) require realistic Monte Carlo simulation of underlying physics

gmc_trans provides Collins and Sivers amplitudes for pions and kaons based on Gaussian Ansatz for TMDs

Positivity limits ➔ smaller Gaussian width for TMDs

Comparison of unpolarized hadron yield suggests $z$-dependent average fragmentation $K_T$

Don’t fully trust GRSV strange polarization at low $Q^2$!

Non-trivial role of unmeasured TMDs in fulfilling positivity of Sivers distribution