Transversity and Beyond at HERMES

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Transversity 2008
Ferrara, May 31st 2008
June 30th, 2007 (around midnight)

Thank You HERA
R.I.P.
Data taking is over, but data analysis by far not!
Inclusive DIS

\[
\frac{d^2 \sigma(s, S)}{dx \, dQ^2} = \frac{2\pi\alpha^2 y^2}{Q^6} \mathbf{L}_{\mu\nu}(s) \mathbf{W}^{\mu\nu}(S)
\]

Lepton Tensor

Hadron Tensor

parametrized in terms of Structure Functions

\[
\frac{d^3 \sigma}{dx dy d\phi} \propto \frac{y}{2} F_1(x, Q^2) + \frac{1 - y - \gamma^2 y^2/4}{2xy} F_2(x, Q^2)
\]

\[
-P_l P_T \cos \alpha \left[ \left( 1 - \frac{y}{2} - \frac{\gamma^2 y^2}{4} \right) g_1(x, Q^2) - \frac{\gamma^2 y}{2} g_2(x, Q^2) \right]
\]

\[
+P_l P_T \sin \alpha \cos \phi \gamma \sqrt{1 - y - \frac{\gamma^2 y^2}{4}} \left( \frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right)
\]
Parton-Model Interpretation of Structure Functions

\[ F_1(x) = \frac{1}{2} \sum_q e_q^2 f_q^1(x) \]
\[ F_2(x) = x \sum_q e_q^2 f_q^1(x) \]
\[ g_1(x) = \frac{1}{2} \sum_q e_q^2 g_q^1(x) \]
\[ g_2(x) = 0 \]

quark-spin contribution to nucleon helicity
Closer Look at $g_2$

- higher-twist, thus
- no probabilistic interpretation
- probes parton correlations

At LO: \[ g_2 = \frac{1}{2} \sum e_q^2 g_T(x) \]

\[ \int_0^1 dx x g_2(x, Q^2) = \frac{1}{3} \left( -a_2(Q^2) + d_2(Q^2) \right) \]

second moment of $g_1$

sensitive to a quark and a gluon amplitude
What HERMES can do for $g_2$

- unfortunately, HERA II with no or only low beam polarization (as compared to HERA I)
  - low figure of merit
  - expected precision not comparable to E155
unfortunately, HERA II with no or only low beam polarization

low figure of merit

expected precision not comparable to E155

What HERMES can do for $g_2$
... back to parton distributions ...
Quark Structure of the Nucleon

Unpolarized quarks and nucleons

Longitudinally polarized quarks and nucleons

Transversely polarized quarks and nucleons

\[ f_1^q(x) : \text{spin averaged (well known)} \]

\[ g_1^q(x) : \text{helicity difference (known)} \]

\[ h_1^q(x) : \text{transversity (hardly known!)} \]

\[ \Rightarrow \text{Vector Charge or Charge} \]

\[ \langle PS|\bar{\Psi}\gamma^\mu\Psi|PS\rangle = \int dx(f_1^q(x) - f_\bar{q}(x)) \]

\[ \langle PS|\bar{\Psi}\gamma^\mu\gamma^5\Psi|PS\rangle = \int dx(g_1^q(x) + g_\bar{q}(x)) \]

\[ \langle PS|\bar{\Psi}\sigma^{\mu\nu}\gamma^5\Psi|PS\rangle = \int dx(h_1^q(x) - h_\bar{q}(x)) \]

Alexei & Co., THANKS!!
SSAs in One-Hadron Production

\[ A_{UT}(\phi, \phi_S) = \frac{1}{\langle |S_{\perp}| \rangle} \frac{N_h^\uparrow(\phi, \phi_S) - N_h^\downarrow(\phi, \phi_S)}{N_h^\uparrow(\phi, \phi_S) + N_h^\downarrow(\phi, \phi_S)} \]

\[ \sim \sin(\phi + \phi_S) \sum_q e_q^2 \mathcal{I} \left[ \frac{k_T \hat{P}_T}{p_T} H_{1}^{1,q} (z, k_{T}^{2}) \right] \]

\[ + \sin(\phi - \phi_S) \sum_q e_q^2 \mathcal{I} \left[ \frac{T P_{h_{\perp}}}{M} \frac{1}{1T} f_{1T}^{1,q} (x, p_{T}^{2}) D_{1}^{q} (z, k_{T}^{2}) \right] \]

\[ \Rightarrow 2D \text{ Max.Likelihood fit to get Collins and Sivers amplitudes:} \]

\[ PDF(2\langle \sin(\phi \pm \phi_S) \rangle_{UT}, \ldots, \phi, \phi_S) = \frac{1}{2} \{ 1 + P_T(2\langle \sin(\phi \pm \phi_S) \rangle_{UT} \sin(\phi \pm \phi_s) + \ldots) \} \]

cf. L. Pappalardo’s talk for results

\[ \mathcal{I}[\ldots]: \text{ convolution integral over initial (}p_T\text{) and final (}k_T\text{) quark transverse momenta} \]
Resolving the Convolution Integral

Weight with transverse hadron momentum $P_{h\perp}$ to resolve convolution:

$$\tilde{A}_{UT}(\phi, \phi_S) = \frac{1}{\langle S_\perp \rangle} \sum_{i=1}^{N^+} P_{h\perp,i} - \sum_{i=1}^{N^-} P_{h\perp,i}$$

$$\sim \sin(\phi + \phi_S) \cdot \sum_q e_q^2 h_{1T}^{q}(x) z H_1^{(1),q}(z) \quad (1): \quad p_T^2/k_T^2$$-moment of distribution / fragmentation function

$$- \sin(\phi - \phi_S) \cdot \sum_q e_q^2 f_{1T}^{(1),q}(x) z D_1^{q}(z)$$

$$+ \ldots$$

- factorized expressions
- $Q^2$ evolution under control
- model-independent analysis of DFs and FFs possible

Gunar Schnell
What about Integration over Transverse Momentum?

Large acceptance effects observed when comparing reconstructed weighted amplitudes in $4\pi$ vs. acceptance.
Extracting full kinematic dependence

1. The full kinematic dependence of the Collins and Sivers moments on 
\( \bar{x} \equiv (x, Q^2, z, P_{h\perp}) \) is evaluated from the real data through a fit of the full 
set of SIDIS events based on a Taylor expansion on \( \bar{x} \):

\[
f(\bar{x}, P_t; c) = 1 + P_t \cdot [A_{Collins}(\bar{x}; c_i) \cdot \sin(\phi + \phi_S) + A_{Sivers}(\bar{x}; c_i) \cdot \sin(\phi - \phi_S)]
\]

e.g.: \( A_{Collins}(\bar{x}, c) = c_0 + c_1 \cdot x + c_2 \cdot z + c_3 \cdot Q^2 + c_4 \cdot P_{h\perp} + c_5 \cdot x^2 + ... + c_{22} \cdot x^2 \cdot z \cdot P_{h\perp} \)

2. The extracted azimuthal moments \( A_{Collins}(\bar{x}; c_i) \) and \( A_{Sivers}(\bar{x}; c_i) \) are folded 
with the spin-independent cross section (known!) in \( 4\pi (\sigma_{UU}^{4\pi}) \) and within the 
HERMES acceptance \( (\sigma_{UU}^{\text{acc.}}) \):

\[
\left\langle \frac{P_{h\perp}}{zM} \sin(\phi \pm \phi_S) \right\rangle_{UT}^{\text{acc.4\pi}}(x) = \frac{\int P_{h\perp} / (zM) \cdot \sigma_{UU}^{\text{acc.4\pi}}(\bar{x}) \cdot A_{Collins,Sivers}(\bar{x}; c_i)}{\int \sigma_{UU}^{\text{acc.4\pi}}(\bar{x})}
\]

L. Pappalardo’s talk at Transversity’08
From SSA amplitudes to TMDs
**Leading-Twist TMDs**

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<th>Nucleon</th>
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**Sivers function**
Using *Purities* to extract PDFs

\[
\widetilde{A}_{UT}^{\sin(\phi-\phi_s),h}(x) = C \cdot \frac{\sum_q e_q^2 f_{1T}^{(1),q}(x) \int dz D_{1}^{q,h}(z) A(x, z)}{\sum_{q'} e_{q'}^2 f_{1}^{q'}(x) \int dz D_{1}^{q',h}(z) A(x, z)}
\]

\[
= C \cdot \sum_q \frac{e_q^2 f_{1}^{q}(x) D_{1}^{q,h}(x)}{\sum_{q'} e_{q'}^2 f_{1}^{q'}(x) D_{1}^{q',h}(x)} \cdot \frac{f_{1T}^{(1),q}}{f_{1}^{q}}(x)
\]

\[
= C \cdot \sum_q P_{q}^{h}(x) \cdot \frac{f_{1T}^{(1),q}}{f_{1}^{q}}(x)
\]

- when using weighted asymmetries, model-independent extraction of PDFs (e.g., Sivers) possible
- used successfully in HERMES helicity-DF analysis
- purities are completely unpolarized objects \( \Rightarrow \) present MC can be used
- Sivers case: easy as the only FF that appears is \( D_1 \)
Purities at HERMES

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{hermes_purities}
\caption{Purity distributions for different quark flavors.}
\end{figure}
Using *Purities* to extract PDFs

\[
\hat{A}_{UT}^{\sin(\phi+\phi_S),h}(x) = C \cdot \frac{\sum_q e_q^2 h_q^1(x) \int dz \, H_{1}^{\perp(1),q,h}(z) A(x, z)}{\sum_{q'} e_{q'}^2 f_{1}^{q'}(x) \int dz \, D_{1}^{q',h}(z) A(x, z)}
\]

\[
= C \cdot \sum_q e_q^2 \frac{f_{1}^{q}(x)}{f_{1}^{q'}(x)} \frac{H_{1}^{\perp(1),q,h}(x)}{D_{1}^{q',h}(x)} \cdot \frac{h_q^1}{f_{1}^{q}(x)}
\]

\[
= C \cdot \sum_q \mathcal{P}_q^h(x) \cdot \frac{h_q^1}{f_{1}^{q}(x)}
\]

- when using weighted asymmetries, model-independent extraction of PDFs (e.g., Sivers) possible
- used successfully in HERMES helicity-DF analysis
- purities are completely unpolarized objects ➔ present MC can be used
- **Collins** case: need to consider Collins FF
Valence-Quark Sivers DF

- look at difference in charged-pion yields: $\Delta N = N_{\pi^+} - N_{\pi^-}$:

$$A_{UT}^{\pi^+-\pi^-}(\phi, \phi_S) \equiv \frac{1}{S_T} \frac{\Delta N^{\uparrow}(\phi, \phi_S) - \Delta N^{\downarrow}(\phi, \phi_S)}{\Delta N^{\uparrow}(\phi, \phi_S) + \Delta N^{\downarrow}(\phi, \phi_S)}$$

- simple interpretation in terms of valence distributions:

$$\langle \sin(\phi - \phi_S) \rangle_{\pi^+-\pi^-}^{UT}(\phi, \phi_S) = -\frac{4f_{\perp,u}^{1T} - f_{\perp,d}^{1T}}{4f_{u}^{1T} - f_{d}^{1T}}$$

at least for weighted asymmetries, but also for unweighted case convolutions simplify
Leading-Twist TMDs

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“Pretzelosity”
Leading-Twist TMDs

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<td>$h_{1T}^\perp$</td>
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the “weird/boost” distributions

Incredible amount of information on the nucleon structure!!!

Exciting time for the new generation experiments!
1-Hadron Production ($ep \rightarrow ehX$)

\[ d\sigma = d\sigma_{UU}^0 + \cos 2\phi d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi d\sigma_{LU}^3 \]

\[ + S_L \left\{ \sin 2\phi d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi d\sigma_{UL}^5 + \lambda_e \left[ d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi d\sigma_{LL}^7 \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) d\sigma_{UT}^8 + \sin(\phi + \phi_S) d\sigma_{UT}^9 + \sin(3\phi - \phi_S) d\sigma_{UT}^{10} \right. \]

\[ \left. + \frac{1}{Q} \left( \sin(2\phi - \phi_S) d\sigma_{UT}^{11} + \sin \phi_S d\sigma_{UT}^{12} \right) \right\} \]

**Collins Effect:**

sensitive to quark transverse spin in “Pretzelicity” distribution or “Boostisity” \[ h_{1L} \]

Bacchetta et al., JHEP 0702 (2007) 093

1-Hadron Production (ep→ehX)

\[
d\sigma = d\sigma_{UU}^0 + \cos 2\phi \, d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi \, d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi \, d\sigma_{LU}^3 \\
+ S_L \left\{ \sin 2\phi \, d\sigma_{UL}^4 \right\} \\
+ S_T \left\{ \sin(\phi - \phi_S) \, d\sigma_{UT}^8 - \frac{1}{Q} \left( \sin(2\phi - \phi_S) \, d\sigma_{UT}^9 + \sin \phi_S \, d\sigma_{UT}^{10} \right) \right\} \\
+ \lambda_e \left[ \cos(\phi - \phi_S) \, d\sigma_{LT}^{13} + \frac{1}{Q} \left( \cos \phi_S \, d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) \, d\sigma_{LT}^{15} \right) \right] \right\}
\]

DSA involving spin-independent FF and “Boostisity” $g_{1T}$

Bacchetta et al., JHEP 0702 (2007) 093
... more twist-3 ...
1-Hadron Production (ep → ehX)

\[
d\sigma = d\sigma^0_{UU} + \cos 2\phi \, d\sigma^1_{UU} + \frac{1}{Q} \cos \phi \, d\sigma^2_{UU} + \lambda_e \frac{1}{Q} \sin \phi \, d\sigma^3_{LU} \\
+S_L \left\{ \sin 2\phi \, d\sigma^4_{UL} + \frac{1}{Q} \sin \phi \, d\sigma^5_{UL} + \lambda_e \left[ d\sigma^6_{LL} + \frac{1}{Q} \cos \phi \, d\sigma^7_{LL} \right] \right\} \\
+\int \sin(\phi - \phi_S) \, d\sigma^8 + \sin(\phi + \phi_S) \, d\sigma^9_{UT} + \sin(3\phi - \phi_S) \, d\sigma^{10}_{UT} \\
+ \lambda_e \left[ \cos(\phi - \phi_S) \, d\sigma^{13}_{LT} + \frac{1}{Q} \left( \cos \phi_S \, d\sigma^{14}_{LT} + \cos(2\phi - \phi_S) \, d\sigma^{15}_{LT} \right) \right] \right\} \\

\text{sensitivity to or needed for transversity}

Bacchetta et al., JHEP 0702 (2007) 093
\(-\mathcal{I}\left[\frac{k_T \cdot p_T}{2MM_h} (xh_T H_1^\perp - xh_T^T H_1^\perp - \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} + \frac{M_h}{M} f_{1T} \frac{\tilde{D}^\perp}{z})\right]\)

\(-h_1 + \tilde{h}_T - \frac{p_T^2}{2M^2x} h_{1T}\)

\(h_1 - \tilde{h}_T^\perp - \frac{p_T^2}{2M^2x} h_{1T}\)

\(-2h_1 + (\tilde{h}_T + \tilde{h}_T^\perp)\)
AUL & Mixing of Azimuthal Moments

Experiment: Target Polarization w.r.t. Beam Direction (l)!

Theory: Polarization along virtual photon direction (q)

⇒ mixing of “experimental” and “theory” asymmetries via:

\[ \begin{pmatrix} \langle \sin \phi \rangle_{UL}^{l} \\ \langle \sin(\phi - \phi S) \rangle_{UT}^{l} \\ \langle \sin(\phi + \phi S) \rangle_{UT}^{l} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\gamma^*} & -\sin \theta_{\gamma^*} & -\sin \theta_{\gamma^*} \\ \frac{1}{2} \sin \theta_{\gamma^*} & \cos \theta_{\gamma^*} & 0 \\ \frac{1}{2} \sin \theta_{\gamma^*} & 0 & \cos \theta_{\gamma^*} \end{pmatrix} \begin{pmatrix} \langle \sin \phi \rangle_{UL}^{q} \\ \langle \sin(\phi - \phi S) \rangle_{UT}^{q} \\ \langle \sin(\phi + \phi S) \rangle_{UT}^{q} \end{pmatrix} \]

\[ \cos \theta_{\gamma^*} \simeq 1, \ \sin \theta_{\gamma^*} \ \text{up to } 15\% \ \text{at HERMES energies} \]
Twist-3 at HERMES

\[ \langle \sin \phi \rangle_{UL}^q = \langle \sin \phi \rangle_{UL}^l + \sin \theta_{\gamma^*} \left( \langle \sin(\phi + \phi_S) \rangle_{UT}^l + \langle \sin(\phi - \phi_S) \rangle_{UT}^l \right) \]

- twist-3 dominates measured asymmetries on longitudinally polarized targets!
- significantly positive for \( \pi^+ \)
- consistent with zero for \( \pi^- \)
- twist-3 not necessarily small

Another Longitudinal SSA: $A_{LU}$

longitudinally pol. beam & unpol. target $\Rightarrow$ subleading-twist

$$\langle \sin \phi \rangle_{LU} \propto \lambda e \frac{M}{Q} \mathcal{I} \left[ xe(x) H_1^+(z) - \frac{M_h}{z M} h_1^+(x) E(z) \right]$$

$\Rightarrow$ for long time candidate to access $e(x)$

$(h_1^+(x)$ contribution either assumed to be zero (T-odd!) or small(??))
Another Longitudinal SSA: $A_{LU}$

longitudinally pol. beam & unpol. target $\Rightarrow$ subleading-twist

$$\langle \sin \phi \rangle_{LU} \propto \lambda_e \frac{M}{Q} I \left[ x e(x) H_1^\perp(z) - \frac{M_h}{z M} h_1^\perp(x) E(z) \right. + \frac{M_h}{z M} f_1(x) G^\perp(z) - x g^\perp(x) D_1(z) + \left. \frac{m_q}{M} h_1^\perp(x) D_1(z) - \frac{m_q}{M} f_1(x) H_1^\perp(z) \right]$$

quark-mass suppressed $\Rightarrow$

Longit. Beam-Spin Asymmetries

- Significantly positive amplitudes for neutral and positive pions
- Much more data on tape:
  - In total factor 7 for H
  - Factor 3 for D
- Mostly with RICH detector, thus kaon amplitudes possible
What can we learn from $A_{UL}$?

$$\langle \sin \phi \rangle_{LU} \propto \lambda e \frac{M}{Q} \int \left[ x e(x) H_1^\perp(z) - \frac{M_h}{z M} h_1^\perp(x) E(z) 
- x g_1^\perp(x) D_1(z) + \frac{M_h}{z M} f_1(x) G^\perp(z) \right]$$

- any help from other observables to separate contributions?
- jet SIDIS $\Rightarrow$ only $g^\perp$-term survives
- 2-hadron production $\neq$ nonzero! (cf. R. Fabbri’s talk)
2-Hadron Fragmentation

- so far: sp interference
  - look at pp interference
- \(\pi K, KK\) pair production
- spin-1 fragmentation (\(\rho^0\))
- ...
- for \(A_{UT}\) and \(A_{LU}\) (the latter also on D target)

\[ H_{1,sp}^{<,sp} \neq 0 \]
now something completely different
SSA in inclusive DIS

- transverse SSA require **interference** of amplitudes with different phases
- achievable via loop diagrams, e.g.
  - Sivers DF includes gauge link (soft gluon exchange)
- How about inclusive DIS?
- 2-photon exchange could provide such mechanism in inclusive DIS

2γ-exchange sensitivity @ HERMES

LR asymmetry of acceptance

even more e^- data available!
The other inclusive SSA

- instead of inclusive DIS, look at inclusive hadron production (a la E704 etc., but photo-production)
- plenty of data available
- can they be related to Sivers effect in any way?
- or what is the physics of Left-Right asymmetries in photo-production?
Conclusions
plenty of projects

inclusive DIS ($g_2$, 2-photon exchange)

inclusive hadron left-right asymmetries

SSA and DSA in semi-inclusive DIS ➞ access to various TMDs like Sivers, Pretzelosity and the “weird ones”

flavor decomposition of Sivers function via purity analysis and pion-yield difference asymmetries

2-hadron fragmentation: sp- & pp-interference etc., in $A_{LU}$ and $A_{UT}$

“only the sky*) is the limit”

*) sky = manpower