The Angular Momentum Structure of the Nucleon

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3-dimensional Picture of the Proton

Nucleon momentum in Infinite Momentum Frame: \((p_{\gamma^*} + p_{\text{nucl}})_z \to \infty\)

- Form factor
- Parton density
- Generalized parton distribution at \(\tau = 0\)

Nucleon’s transv. charge distribution given by 2-dim. Fourier transform of **Form Factor**:

\(\Rightarrow\) Parton’s transverse localization \(b_\perp\)

Probability density to find partons of given long. mom. fraction \(x\) at resol. scale \(1/Q^2\)

(no transv. inform.)

\(\Rightarrow\) Parton’s longitudinal momentum distribution function (PDF) \(f(x)\)

**Generalized Parton Distributions (GPDs)** probe simultaneously transverse localization \(b_\perp\)

for a given longitudinal momentum fraction \(x\).

2nd moment by Ji relation:

\[
J_{q,g} = \frac{1}{2} \lim_{t \to 0} \int x \, dx \\
[H_{q,g}(x, \xi, t) + E_{q,g}(x, \xi, t)]
\]
Proton Spin Budget in a Nutshell

NO unique and gauge-invariant decomposition of the nucleon spin:

(A) ‘GPD-based’:
\[ \frac{1}{2} = J_q + J_g = \frac{1}{2} \Delta \Sigma + L_q + \hat{\Delta} g + L_g \]

- Total angular momenta of quarks \( J_q \) and gluons \( J_g \) are gauge-invariant and calculable in lattice gauge theory
- Intrinsic spin contribution and orbital angular momentum are gauge inv. for quarks (\( \frac{1}{2} \Delta \Sigma \) and \( L_q \)), but not for gluons (\( \hat{\Delta} g \) and \( L_g \))
- Probabilistic interpretation only for \( \frac{1}{2} \Delta \Sigma \) (well measured)
- \( J_q \) accessible through exclusive lepton nucleon scattering
- \( J_g \) very difficult to access experimentally

(B) Light-cone gauge:
\[ \frac{1}{2} = \mathcal{J}_q + \mathcal{J}_g = \frac{1}{2} \Delta \Sigma + \mathcal{L}_q + \Delta g + \mathcal{L}_g \]

- All 4 terms have a probabilistic interpretation
- \( \Delta g \) is gauge invariant (being measured)

⇒ Results from both decompositions must not be mixed, as
\( \mathcal{L}_q \neq L_q, \Delta g \neq \hat{\Delta} g, \mathcal{L}_g \neq L_g, \) even \( \mathcal{J}_g \neq J_g \)!
DIS: Kinematics, Cross Sections, Asymmetry

Virtual-photon kinematics:
\[ Q^2 = -q^2 \quad \nu = E - E' \]

Fraction of nucleon momentum carried by struck quark:
\[ x = \frac{Q^2}{2M \nu} \]

Fraction of virtual-photon energy carried by produced hadron \( h \):
\[ z = \frac{E_h}{\nu} \]

Hadron transverse momentum:
\[ P_{h\perp} \]

Unpolarized cross section:
\[ \sigma_{UU} \equiv \frac{1}{2}(\sigma_{\leftarrow} + \sigma_{\rightarrow}) \]

Cross section (helicity) difference:
\[ \sigma_{LL} \equiv \frac{1}{2}(\sigma_{\leftarrow} - \sigma_{\rightarrow}) \]

Double-spin asymmetry:
\[ A_{||} \equiv \frac{\sigma_{LL}}{\sigma_{UU}} \simeq \frac{g_1}{F_1} \] (neglecting small \( g_2 \) contribution)

Measured asymmetry:
\[ A_{||} = \frac{1}{\langle P_B \rangle \langle P_T \rangle} \frac{(\frac{N}{L})_{\leftarrow} - (\frac{N}{L})_{\rightarrow}}{(\frac{N}{L})_{\leftarrow} + (\frac{N}{L})_{\rightarrow}} \]

with \( P_B(P_T) \): longitudinal beam (target) polarization
Direct determination of quark spin contribution $\Delta \Sigma$

Most precise $g_1^d$ result: Hermes inclusive data [PRD75(2007)012007,hep-ex/0609039]:

Method:
- NNLO leading twist analysis in $\overline{\text{MS}}$ scheme
- assume SU$_3$ flavor symmetry in hyperon decay
- observe saturation of $\Gamma_1 = \int dx \ g_1^d(x)$ for $x < 0.04$
- assume no significant contribution of small-$x$ region

Data for $Q^2 > 1 \text{ GeV}^2$:
- evaluate $\Gamma_1^d(Q^2 = 5 \text{ GeV}^2) = 0.021 \int_{0.9}^{0.1} dx \ g_1^d(x)$

Result at $Q^2 = 5 \text{ GeV}^2$ (all data points evolved):
$$\Delta \Sigma = 0.330 \pm 0.011_{\text{theor.}} \pm 0.025_{\text{exp.}} \pm 0.028_{\text{evol.}}$$
where ‘exp.’ includes stat., syst. and parameterization uncertainties
**Next-to-leading Order QCD Fits**

**Results by AAC** [PRD74(2006)014015,hep-ph/0603213]: NLO in $\alpha_s$, $\overline{MS}$ scheme

**Assumptions:**
- Flavor-symmetric $\Delta q_{sea}$
- Integrals of $\Delta q_{val}^u$ and $\Delta q_{val}^d$ fixed by weak decay constants $F$ and $D$

**Input experimental data:**
- $A_{1p,d}^A$ from COMPASS, JLAB, HERMES
- $A_{LL}^{\pi^0}$ from PHENIX

**Results at $Q^2 = 1$ GeV$^2$:**
- $\Delta \Sigma = 0.25 \pm 0.10$
- $\Delta G = 0.47 \pm 1.08$ (DIS alone)
- $\Delta G = 0.31 \pm 0.32$ (DIS+PHENIX)

**Impact of recent CLAS and COMPASS data** [PRD75(2007)074027,hep-ph/0612360]:
- Fit with $\Delta g > 0$ : $\Delta G = 0.13 \pm 0.17$
- Fit with $\Delta g < 0$ : $\Delta G = -0.20 \pm 0.41$

**Impact of recent PHENIX and STAR data** ($Q^2 = 10$ GeV$^2$) [DSSV, arXiv:0804.0422 [hep-ph]]:
- Clear indication for flavor-asymmetric sea.
- For $0 < x < 1$: $\Delta G = -0.084$
- For $0.001 < x < 1$: $\Delta G = 0.013$ with $^{+0.106}_{-0.120}$ for $\Delta \chi^2 = 1$; $^{+0.702}_{-0.314}$ for $\Delta \chi^2 / \chi^2 = 2\%$

Wolf-Dieter Nowak, Perspectives in Hadronic Physics, May 15, 2008 – p. 7
Determination of Gluon Contribution to Nucleon Spin

- **High-\(p_t\) hadron pairs or single hadrons quasi-real photoprod.:** \(<Q^2> \approx 0.1 \text{ GeV}^2\)
- **Sensitivity through** \(\gamma^* g\) ‘direct’ hard scattering or ‘resolved-photon’ process
  - left graphs: direct processes; right graphs: resolved-photon processes [COMPASS analysis]

Extraction heavily relies on **PYTHIA** simulation (LO only !)

**Processes:**
- **COMPASS:** Open-charm production (\(\gamma^* g \rightarrow c\bar{c}\)) and hadron pairs
- **HERMES:** Single high-\(p_t\) hadrons. Pairs in old analysis (all \(Q^2\), \(<x_g> \approx 0.17\)
  - **[PRL84 (2000) 2584]** \(\frac{\Delta g}{g} = 0.41 \pm 0.18_{stat} \pm 0.03_{sys-exp} (\pm \text{unknown}_{sys-Model})\)
- **RHIC:** \(A_{LL}\) in inclusive direct \(\gamma\) & \(\pi^0\) production, inclusive jet production
Results on Gluon Helicity Distribution $\frac{\Delta g}{g}(x)$

**DIS results on $\frac{\Delta g}{g}(x)$:**

**COMPASS high-$p_t$ hadron pairs:**

$Q^2 < 1 \text{ GeV}^2 (\langle x \rangle \simeq 0.085)$:

$\frac{\Delta g}{g} = 0.016 \pm 0.058_{\text{stat}} \pm 0.055_{\text{syst}}$

{PLB 612,154 (2005)}

$Q^2 > 1 \text{ GeV}^2 (\langle x_g \rangle \simeq 0.13)$

$\frac{\Delta g}{g} = 0.06 \pm 0.31_{\text{stat}} \pm 0.06_{\text{syst}}$

{prel.: K.Kurek,DIS06,hep-ex/0607061}

**COMPASS open charm:**

$\frac{\Delta g}{g} = -0.47 \pm 0.44_{\text{stat}} \pm 0.15_{\text{syst}}$

$(\langle x_g \rangle \simeq 0.11)$ {arXiv:0802.3023[hep-ex]}

**HERMES high-$p_t$ single hadrons [prel.]:**

$Q^2 \simeq 0; (\langle x_g \rangle \simeq 0.22)$: $\frac{\Delta g}{g} = 0.071 \pm 0.034_{\text{stat}} \pm 0.010_{\text{sys-exp}} \pm 0.127_{0.105} \text{ sys-Models}$

**PHENIX:** Confidence limits for fits with different $\frac{\Delta g}{g}$ assumptions
Deeply Virtual Compton Scattering

Same final state in **DVCS** and Bethe-Heitler ⇒ Interference!
\[ d\sigma(eN \rightarrow eN\gamma) \propto |T_{BH}|^2 + |T_{DVCS}|^2 + T_{BH}T_{DVCS}^* + T_{BH}^*T_{DVCS} \]

- \( T_{BH} \) is parameterized in terms of Dirac and Pauli Form Factors \( F_1, F_2 \), calculable in QED.
- \( T_{DVCS} \) is parameterized in terms of Compton form factors \( \mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}} \) (which are convolutions of resp. GPDs \( H, E, \tilde{H}, \tilde{E} \))
- (Certain Parts of) interference term \( I \) can be filtered out by forming certain cross section differences (or asymmetries)
  ⇒ GPDs \( H, E, \tilde{H}, \tilde{E} \) indirectly accessible via interference term \( I \)
Azimuthal Asymmetries in DVCS

DVCS–Bethe-Heitler Interference term $I_i$ induces differences or azimuthal asymmetries $\mathcal{A}$ in the measured cross-section:

- **Beam-charge asymmetry** $\mathcal{A}_{C}(\phi)$ [BCA]:
  \[ d\sigma(e^+, \phi) - d\sigma(e^-, \phi) \propto \text{Re}[F_1\mathcal{H}] \cdot \cos \phi \]

- **Beam-spin asymmetry** $\mathcal{A}_{LU}(\phi)$ [BSA]:
  \[ d\sigma(\vec{e}, \phi) - d\sigma(\vec{e}, \phi) \propto \text{Im}[F_1\mathcal{H}] \cdot \sin \phi \]

- **Long. target-spin asymmetry** $\mathcal{A}_{UL}(\phi)$:
  \[ d\sigma(P, \phi) - d\sigma(P, \phi) \propto \text{Im}[F_1\tilde{\mathcal{H}}] \cdot \sin \phi \] [LTSA]

- **Transverse target-spin asymmetry** $\mathcal{A}_{UT}(\phi, \phi_S)$ [TTSA]:
  \[ d\sigma(\phi, \phi_S) - d\sigma(\phi, \phi_S + \pi) \propto \text{Im}[F_2\mathcal{H} - F_1\mathcal{E}] \cdot \sin (\phi - \phi_S) \cos \phi \]
  \[ + \text{Im}[F_2\tilde{\mathcal{H}} - F_1\xi\tilde{\mathcal{E}}] \cdot \cos (\phi - \phi_S) \sin \phi \]

($F_1, F_2$ are the Dirac and Pauli elastic nucleon form factors)
Various asymmetry amplitudes $A$ contribute to polarized cross section $\sigma_{LU}$:

$$\sigma_{LU}(\phi; P_l, e_l) = \sigma_{UU}(\phi)[1 + e_l A_C(\phi) + e_l P_l A_{LU}^I(\phi) + P_l A_{LU}^{DVCS}(\phi)]$$

$L$: longitudinally polarized lepton beam of charge $e_l$ & polarization $P_l$; $U$: unpolarized proton target

BCA:

$$A_C(\phi) = \frac{1}{\sigma_{UU}} c_1^I \cos \phi + \cdots \quad c_1^I \propto \frac{\sqrt{-t}}{Q} F_1 \text{Re}\mathcal{H} + [\cdots]$$

BSA (interference term):

$$A_{LU}^I(\phi) = \frac{1}{\sigma_{UU}} s_1^I \sin \phi + \cdots \quad s_1^I \propto \frac{\sqrt{-t}}{Q} F_1 \text{Im}\mathcal{H} + [\cdots]$$

BSA (DVCS term):

$$A_{LU}^{DVCS}(\phi) = \frac{1}{\sigma_{UU}} s_{DVCS}^I \sin \phi \quad \text{(small at HERMES energy)}$$

Unpolarized cross section: $\sigma_{UU} = \sigma_{BH} + \sigma_{DVCS} + \sigma_I$

$F_1$: Dirac elastic nucleon form factor

$\mathcal{H}$: Compton Form Factor (CFF), embodies GPD $H$

$[\cdots]$: kinematically suppressed CFFs ($\tilde{\mathcal{H}}, \mathcal{E}$) embodying GPDs $\tilde{H}, E$

Fit to data:

$$A_C(\phi) = \sum_{n=0}^{3} A_C^{\cos n\phi} \cos n\phi$$
$$A_{LU}^I(\phi) = \sum_{m=1}^{2} A_{LU,I}^{\sin m\phi} \sin m\phi$$
$$A_{LU}^{DVCS}(\phi) = A_{LU,DVCS}^{\sin \phi} \sin \phi$$

Fit results: ‘effective’ asymmetry amplitudes: $A_C^{\cos n\phi}, A_{LU,I}^{\sin m\phi}, A_{LU,DVCS}^{\sin \phi}$

$\Rightarrow$ well defined in theory, can be compared to GPD models !
HERMES Combined BSA & BCA Results

**BSA**
- const.term

**BCA**
- $\propto F_1 \text{Im}\mathcal{H}$
- $\propto -A_C^{\cos \phi}$
- $\propto F_1 \text{Re}\mathcal{H}$

Overall Results:

- Res. frac
- $A_C \cos \phi$
- $A_C \cos 2\phi$
- $A_C \cos 3\phi$

**HT**

$0 < -t < 0.7$  
$0.03 < x_B < 0.35$  
$1 < Q^2 < 10$
Discussion of Combined BSA & BCA Analysis

!!! Asymmetries of ‘associated (resonance) production’ are unknown !!!

Kinematic dependence of fractions of associated production known from MC:

Average is 12%

⇒ In data associated production is part of the signal, while in models it is not included (still unknown)

- HERMES BSA agrees with Dual model Guzey, (Polyakov), Teckentrup 2006
- HERMES BCA disfavours factorized $t$ dep., in both models and D-term in VGG
- Pure $|DVCS|^2$ asymmetries found compatible with zero (as models assume)

⇒ HERMES data precise enough to discriminate between models or their variants
⇒ new models eagerly awaited !!! Müller, Kumericki
Why TTSA Data Expected to be Sensitive to $J_q$?

$A_{UT}(\phi, \phi_S) \propto \text{Im}[F_2H - F_1E] \sin(\phi - \phi_S) \cos \phi + \text{Im}[\tilde{F}_2\tilde{H} - F_1\tilde{E}] \cos(\phi - \phi_S) \sin \phi$

**ANSATZ:** spin-flip Generalized Parton Distribution $E$ is parameterized as follows:

- Factorized ansatz for spin-flip quark GPDs: $E_q(x, \xi, t) = \frac{E_q(x, \xi)}{(1-t/0.71)^2}$
- $t$-indep. part via double distr. ansatz: $E_q(x, \xi) = E_q^{DD}(x, \xi) - \theta(\xi - |x|) D_q \left( \frac{x}{\xi} \right)$
- using double distr. $K_q$: $E_q^{DD}(x, \xi) = \int_{-1}^{-1-|\beta|} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) K_q(\beta, \alpha)$
- with $K_q(\beta, \alpha) = h(\beta, \alpha) e_q(\beta)$ and $e_q(x) = A_q q_{val}(x) + B_q \delta(x)$ based on chiral QSM
- where coeff.s $A, B$ constrained by Ji relation, and $\int_{-1}^{-1} dx e_q(x) = \kappa_q$  
- $A_u, A_d, B_u, B_d$ are functions of $J_u, J_d$
  $\Rightarrow J_u, J_d$ are free parameters when calculating TTSA
- Sensitivity to $J_u$ (with $J_d = 0$) studied [EPJ C46, 729 (2006), hep-ph/0506264]
HERMES: First Measurement of TTSA

\[ A_{UT}(\phi, \phi_S) = A_{UT}^{\sin(\phi-\phi_S)} \cos \phi \cdot \sin (\phi - \phi_S) \cos \phi + A_{UT}^{\cos(\phi-\phi_S)} \sin \phi \cdot \cos (\phi - \phi_S) \sin \phi + \ldots \]
Model-dependent constraints on $J_u$ vs $J_d$

HERMES analysis method:
[arXiv:0802.2499, subm. to JHEP]

Unbinned maximum likelihood fit
to all possible azimuthal asymmetry amplitudes at average kinematics:
⇒ ‘combined fit’ of HERMES BCA and TTSA data against various model calculations, leaving $J_u$ and $J_d$
as free parameters ⇒ model-dep.

1-σ constraints on $J_u$ vs. $J_d$:

- **Double-distribution model:** $J_u + J_d/2.8 = 0.49 \pm 0.17 (\text{exp}_{\text{tot}})$
- **Dual model** [Guzey, Teckentrup]: $J_u + J_d/2.8 = -0.02 \pm 0.27 (\text{exp}_{\text{tot}})$
- **Lattice gauge theory:** QCDSF [Göckeler et al.], LHPC [Hägler et al.]
- **DFJK model:** zero-skewness GPDs extracted from nuclear form factor data using valence-quark contributions only [Diehl et al.]
Summary and Outlook

- No unique and gauge-invariant decomposition of the nucleon spin
- **HERMES** and **COMPASS** results on Deep Inelastic Scattering yield intrinsic quark and gluon contribution to the nucleon spin (in light-cone gauge)
- Total angular momenta of quarks and gluons accessible in context of Generalized Parton Distributions
- Deeply Virtual Compton Scattering is prime candidate to constrain total quark angular momenta (no feasible approach known for gluons)
- Pioneering **HERMES** results on azimuthal asymmetries, and first promising **JLAB** results on cross section differences in DVCS, allow us to severely constrain GPD models
- Increasing theoretical activities on improved and new GPD models
- Short-term future: for DVCS and other exclusive reactions final **HERMES** results and many more very precise **JLAB** 6 GeV data expected
- Medium-term future: hopefully unique **COMPASS** BCA data, presumably many very precise **JLAB** 12 GeV data
5.75 GeV $e^-$ beam (76% pol.), unpol. LH$_2$ target, [PRL 97 (2006) 262002]

Detect $e'$ by HRS, $\gamma$ by EM calorimeter, recoil $p$ by scintillator array

3 different kinematic settings with $x_{Bj} = 0.36$ fixed:
$Q^2 = 1.5, 1.9, 2.3$ GeV$^2$. For each: $-t = 0.17, 0.23, 0.28, 0.33$ GeV

Measured separately: $\frac{d^4 \Sigma}{d^4 \Phi} = \frac{1}{2} \left[ \frac{d^4 \sigma^+}{d^4 \Phi} - \frac{d^4 \sigma^-}{d^4 \Phi} \right]$ and $\frac{d^4 \sigma}{d^4 \Phi} = \frac{1}{2} \left[ \frac{d^4 \sigma^+}{d^4 \Phi} + \frac{d^4 \sigma^-}{d^4 \Phi} \right]$

⇒ distinct information on GPDs:
$\frac{d^4 \Sigma}{d^4 \Phi} \propto \text{Im } I$: as in BSA numer.
$\frac{d^4 \sigma}{d^4 \Phi} \propto \text{Re } I$: same as in BCA

Fit following terms separately:
$|BH^2|$ (dot-dot-dashed),
twist-2 int. term (dashed),
twist-3 int. term (dot-dashed)
($|DVCS|^2$ found below few %)

Twist-3 terms small

$\frac{d^4 \sigma}{d^4 \Phi} > |BH^2| \rightarrow$ BSA and Im $I/|BH^2|$ are not exactly the same over $\Phi$
CLAS E01-113: High-stat. Beam-spin Asymmetry

- 1st dedicated Hall-B DVCS exp’t: 5.76 GeV $e^-$ beam, pol. 76-82%; unpol. LH$_2$ target
- CLAS spectrometer upgraded by inner calorimeter to detect $\gamma$’s at small angles → all 3 final state particles ($e' N \gamma$) detected!
- Broad kinematic coverage at medium $x$ (0.1...0.5), combined with high lumi → 3-dim. binning possible. Unpublished (White Paper) preview:

⇒ Very promising first glimpse into statistical power of JLab DVCS measurements