Single-spin asymmetries in SIDIS off transversely polarized protons at HERMES

Luciano Pappalardo
pappalardo@fe.infn.it
The nucleon spin structure
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<table>
<thead>
<tr>
<th>Quark</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
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<tbody>
<tr>
<td>Nucleon</td>
<td>( f_1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nucleon</td>
<td>( g_1 )</td>
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<tr>
<td>Nucleon</td>
<td></td>
<td></td>
<td>( h_1 )</td>
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The nucleon spin structure

Can be studied by measuring azimuthal asymmetries in SIDIS

<table>
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<td><strong>U</strong></td>
<td>$f_1$</td>
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<td>$h_1$</td>
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<tr>
<td><strong>L</strong></td>
<td>$g_1$</td>
<td></td>
<td>$h_{1L}$</td>
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<tr>
<td><strong>T</strong></td>
<td>$f_{1T}$</td>
<td>$g_{1T}$</td>
<td>$h_1$</td>
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<td></td>
<td></td>
<td></td>
<td>$h_{1T}$</td>
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DF exhibits asymmetries in the azimuthal angles $\phi$ and $\phi_s$. 

$\sigma^{ep \rightarrow ehX}$
Factorization theorem:

\[ \sigma^{ep\rightarrow ehX} = \sum_q DF \times \sigma^{eq\rightarrow eq} \times FF \]

exhibits asymmetries in the azimuthal angles \( \phi \) and \( \phi_s \)
\[ d\sigma = d\sigma_{UU}^0 + \cos 2\phi \ d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi \ d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma_{LU}^3 \]

\[ + S_L \left\{ \sin 2\phi \ d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi \ d\sigma_{UL}^5 + \lambda_e \left[ d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi \ d\sigma_{LL}^7 \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \ d\sigma_{UT}^8 + \sin(\phi + \phi_S) \ d\sigma_{UT}^9 + \sin(3\phi - \phi_S) \ d\sigma_{UT}^{10} \right. \]

\[ + \frac{1}{Q} \sin(2\phi - \phi_S) \ d\sigma_{UT}^{11} + \frac{1}{Q} \sin \phi_S d\sigma_{UT}^{12} \]

\[ + \lambda_e \left[ \cos(\phi - \phi_S) \ d\sigma_{LT}^{13} + \frac{1}{Q} \cos \phi_S d\sigma_{LT}^{14} + \frac{1}{Q} \cos(2\phi - \phi_S) d\sigma_{LT}^{15} \right] \right\} \]
\[ d\sigma = d\sigma_0 + \cos 2\phi \ d\sigma_1 + \frac{1}{Q} \cos \phi \ d\sigma_2 + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma_3 \]

\[ + S_L \left\{ \sin 2\phi \ d\sigma_4 + \frac{1}{Q} \sin \phi \ d\sigma_5 + \lambda_e \left[ d\sigma_6 + \frac{1}{Q} \cos \phi \ d\sigma_7 \right] \right\} \]

\[ + S_T \left\{ \sin(\phi - \phi_S) \ d\sigma_8 + \sin(\phi + \phi_S) \ d\sigma_9 + \sin(3\phi - \phi_S) \ d\sigma_{10} \right\} \]

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\[ \sigma_{BT}^{ep \rightarrow ehX} = \sum_q \sigma^{eq \rightarrow eq} \otimes DF \otimes FF \]

<table>
<thead>
<tr>
<th>Beam pol.</th>
<th>Target pol.</th>
<th>Formula</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>UU</td>
<td>1</td>
<td>[\cos(2\phi_h^l)]</td>
<td>(f_1 = \bullet)</td>
<td>(h_1^\perp = \bullet)</td>
<td>(D_1 = \bullet)</td>
<td>(H_1^\perp = \bullet)</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>UL</td>
<td>(\sin(2\phi_h^l))</td>
<td>(h_{1L}^\perp = \bullet)</td>
<td>(H_1^\perp = \bullet)</td>
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<tr>
<td>UT</td>
<td>(\sin(\phi_h^l + \phi_S^l))</td>
<td>(h_1 = \bullet)</td>
<td>(H_1^\perp = \bullet)</td>
<td>(D_1 = \bullet)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>UT</td>
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<tr>
<td>LL</td>
<td>1</td>
<td>(g_1 = \bullet)</td>
<td>(D_1 = \bullet)</td>
<td></td>
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<tr>
<td>LT</td>
<td>(\cos(\phi_h^l - \phi_S^l))</td>
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\[ \sigma_{\text{BT}}^{ep \to ehX} = \sum_{q} \, \text{DF} \otimes \sigma^{eq \to eq} \otimes \text{FF} \]
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Transversity

\[ \delta q(x, Q^2) = q^\uparrow - q^\downarrow \]

Difference of probabilities to find quarks with spin aligned or anti-aligned to the nucleon transverse spin

Chiral-odd

requires spin flip of the quark

Not measurable in inclusive DIS

Unmeasured for long time!
**Transversity**

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**Sivers function**

\[ f_{1T}^{\perp q}(x, p_T^2) \]

Chiral-even \ T- odd

Probability to find unpolarized quarks with transverse momentum \( p_T \) in a transversely pol. nucleon.

Not measurable in inclusive DIS

Unmeasured for long time!

describes spin-orbit correlation in the nucleon

Requires non-zero orbital angular momentum!

azimuthal asymmetries in the direction of the outgoing hadrons.
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Probability to find unpolarized quarks with transverse momentum \( p_T \) in a transversely pol. nucleon.

**Collins function**

\[ H_1^{\perp}(z, k_T^2) \]

Chiral-odd   T- odd

Correlation between transverse spin of the fragmenting quark and transverse momentum of the produced hadron

**Chiral-odd** requires spin flip of the quark

Not measurable in inclusive DIS

Unmeasured for long time!

Describes spin-orbit correlation in the nucleon

Requires non-zero orbital angular momentum!

Azimuthal asymmetries in the direction of the outgoing hadrons.

Analyzer of fragmenting quark’s transv. polarization

Azimuthal asymmetries in the direction of the outgoing hadrons.
\[ d\sigma = d\sigma_0^U + \cos 2\phi \; d\sigma_1^U + \frac{1}{Q} \cos \phi \; d\sigma_2^U + \lambda_e \frac{1}{Q} \sin \phi \; d\sigma_3^U \]
\[ + S_L \left\{ \sin 2\phi \; d\sigma_4^U + \frac{1}{Q} \sin \phi \; d\sigma_5^U + \lambda_e \left[ d\sigma_6^U + \frac{1}{Q} \cos \phi \; d\sigma_7^U \right] \right\} \]
\[ + S_T \left\{ \sin(\phi - \phi_S) \; d\sigma_8^U + \sin(\phi + \phi_S) \; d\sigma_9^U + \sin(3\phi - \phi_S) \; d\sigma_{10}^U \right. \]
\[ + \frac{1}{Q} \sin(2\phi - \phi_S) \; d\sigma_{11}^U + \frac{1}{Q} \sin \phi_S \; d\sigma_{12}^U \]
\[ + \lambda_e \left[ \cos(\phi - \phi_S) \; d\sigma_{13}^L + \frac{1}{Q} \cos \phi_S \; d\sigma_{14}^L + \frac{1}{Q} \cos(2\phi - \phi_S) \; d\sigma_{15}^L \right] \]

\[ d\sigma_{Sivers}^{obs} \propto |S_T| \sin(\phi - \phi_S) \sum_q e^2_q I \left[ \frac{\vec{p}_T \cdot \hat{p}_{h\perp}}{M_h} f_{1\perp q}(x, p_T^2) \otimes D_i^q(z, k_T^2) \right] \]

\[ d\sigma_{Collins}^{obs} \propto |S_T| \sin(\phi + \phi_S) \sum_q e^2_q I \left[ \frac{\vec{k}_T \cdot \hat{p}_{h\perp}}{M_h} h_1(x, p_T^2) \otimes H_{1\perp q}^q(z, k_T^2) \right] \]

\[ I[... \right] = \text{convolution integral over intrinsic (}\vec{p}_T\text{) and fragmentation (}\vec{k}_T\text{) transverse momenta} \]
\[
\begin{align*}
d\sigma &= d\sigma_{UU}^0 + \cos 2\phi \ d\sigma_{UU}^1 + \frac{1}{Q} \cos \phi \ d\sigma_{UU}^2 + \lambda_e \frac{1}{Q} \sin \phi \ d\sigma_{LU}^3 \\
&\quad + S_L \left\{ \sin 2\phi \ d\sigma_{UL}^4 + \frac{1}{Q} \sin \phi \ d\sigma_{UL}^5 + \lambda_e \left[ d\sigma_{LL}^6 + \frac{1}{Q} \cos \phi \ d\sigma_{LL}^7 \right] \right\} \\
&\quad + S_T \left\{ \sin(\phi - \phi_S) \ d\sigma_{UT}^8 + \sin(\phi + \phi_S) \ d\sigma_{UT}^9 + \sin(3\phi - \phi_S) \ d\sigma_{UT}^{10} \\
&\quad \quad \quad \quad \quad \quad + \frac{1}{Q} \sin(2\phi - \phi_S) \ d\sigma_{UT}^{11} + \frac{1}{Q} \sin \phi_S d\sigma_{UT}^{12} \\
&\quad \quad \quad \quad \quad \quad + \lambda_e \left[ \cos(\phi - \phi_S) \ d\sigma_{LT}^{13} + \frac{1}{Q} \cos \phi_S d\sigma_{LT}^{14} + \frac{1}{Q} \cos(2\phi - \phi_S) d\sigma_{LT}^{15} \right] \right\} \\
\end{align*}
\]

\[
d\sigma_{UT}^{Sivers} \propto |S_T| \sin(\phi - \phi_S) \sum_q e_q^2 I \left[ \frac{\vec{p}_T \cdot \hat{p}_{h\perp}}{M_h} \ f_{1T}^q(x, p_T^2) \otimes D_i^q(z, k_T^2) \right]
\]

Two distinctive signatures if \( \phi_S \neq 0 \) (transversely polarized target)

\[
d\sigma_{UT}^{Collins} \propto |S_T| \sin(\phi + \phi_S) \sum_q e_q^2 I \left[ \frac{\vec{k}_T \cdot \hat{p}_{h\perp}}{M_h} \ h_1^i(x, p_T^2) \otimes H_{1T}^q(z, k_T^2) \right]
\]

\( I[...] \) = convolution integral over intrinsic (\( \vec{p}_T \)) and fragmentation (\( \vec{k}_T \)) transverse momenta
TRD, Calorimeter, preshower, RICH: lepton-hadron > 98%

π ~ 98%, K ~ 88%, P ~ 85%

hadron separation

Aerogel n=1.03

C₄F₁₀ n=1.0014
### Full HERMES transverse data set (2002-2005)

(Transversely polarized hydrogen target: $\langle P \rangle \approx 73\%$

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<th>Semi-inclusive DIS</th>
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<tr>
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</tr>
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The selected SIDIS events are used to extract the Collins and Sivers amplitudes through a Maximum Likelihood fit using the PDF:

$$L = \prod_i (F_i)^{w_i}$$

$$F_i \left( \langle \sin(\phi \pm \phi_S) \rangle_{UT}^h , P_i, \phi, \phi_S \right) \propto 1 + P_i \left[ 2\langle \sin(\phi + \phi_S) \rangle_{UT}^h \sin(\phi + \phi_S) + 2\langle \sin(\phi - \phi_S) \rangle_{UT}^h \sin(\phi - \phi_S) \\ + 2\langle \sin(\phi_S) \rangle_{UT}^h \sin(\phi_S) + 2\langle \sin(3\phi - \phi_S) \rangle_{UT}^h \sin(3\phi - \phi_S) + 2\langle \sin(2\phi - \phi_S) \rangle_{UT}^h \sin(2\phi - \phi_S) \right]$$
Results and interpretation
Collins moments for pions (2002-2005)

- positive amplitude for $\pi^+$
- $\sim 0$ amplitude for $\pi^0$
- negative amplitude for $\pi^-$

\[
\begin{cases}
u \Rightarrow \pi^+ ; \ d \Rightarrow \pi^- \ (fav) \\ 
u \Rightarrow \pi^- ; \ d \Rightarrow \pi^+ \ (unfav)
\end{cases}
\]

the large negative $\pi^-$ amplitude suggests disfavored Collins function with opposite sign:

\[
H_{1,unfav}^\perp (z) \approx -H_{1,fav}^\perp (z)
\]

→ measurement at $e^+e^-$ collider machines

\[
\sim I[h_1^q(x)H_{1,q}^\perp (z)]
\]
Collins moments for pions (2002-2005)

- positive amplitude for $\pi^+$
- $\sim 0$ amplitude for $\pi^0$
- negative amplitude for $\pi^-$

\[
\begin{align*}
&\begin{cases}
  u \Rightarrow \pi^+ 
  ;
  d \Rightarrow \pi^- \text{ (fav)} \\
  u \Rightarrow \pi^- 
  ;
  d \Rightarrow \pi^+ \text{ (unfav)}
\end{cases} \\
\text{the large negative } \pi^- \text{ amplitude suggests disfavored Collins function with opposite sign:}
\end{align*}
\]

\[
H_{1,\text{unfav}}(z) \approx -H_{1,\text{fav}}(z)
\]

- measurement at $e^+e^-$ collider machines

\[\propto I[h_1(x)H_{1,q}(z)] \neq 0\] Transversity & Collins FF $\neq 0$
Collins moments: Pion-kaon comparison

- $K^+$ and $\pi^+$ amplitudes consistent (u-quark dominance)
- $K^-$ and $\pi^-$ amplitudes with opposite sign (but $K^- (\bar{u}s)$ originates from fragmentation of sea quarks)
2-D Collins moments for $\pi^\pm$

$X$ vs. $Z$

HERMES PRELIMINARY 2002-2005
Lepton Beam Asymmetries — 8.1% scale uncertainty
2-D Collins moments for $\pi^\pm$

$X$ vs. $Z$

$X$ vs. $P_{h\perp}$
Sivers moments for pions (2002-2005)

- Positive amplitude for $\pi^+$
- Positive amplitude for $\pi^0$
- Amplitude $\sim 0$ for $\pi^-$

$\propto I[f_{1T}^q(x)D_1^q(z)] \neq 0$  \quad Sivers function $\neq 0$ \quad $L_q \neq 0$
Sivers moments: Pion-kaon comparison

- **K**+ amplitude is larger than for **π**+ conflicts with usual expectations based on u-quark dominance

\[ \pi^+ \equiv (u, \bar{d}) \quad K^+ \equiv (u, \bar{s}) \]

suggests substantial magnitudes of the Sivers function for the sea quarks

- Both **K**− and **π**− amplitudes are consistent with zero
2-D Sivers moments for $\pi^\pm$

$X$ vs. $Z$

$X$ vs. $P_{h\perp}$

HERMES PRELIMINARY 2002-2005

Lepton Beam Asymmetries — 8.1 % scale uncertainty

$0.023 < x < 0.05$

$0.05 < x < 0.06$

$0.09 < x < 0.15$

$0.15 < x < 0.22$

$0.22 < x < 0.40$

$Q^2 = 1.3\text{ GeV}^2$

$Q^2 = 1.9\text{ GeV}^2$

$Q^2 = 2.6\text{ GeV}^2$

$Q^2 = 4.2\text{ GeV}^2$

$Q^2 = 0.2\text{ GeV}^2$

$0.20 < z < 0.30$

$0.30 < z < 0.40$

$0.40 < z < 0.50$

$0.50 < z < 0.60$

$0.60 < z < 0.70$

$2\langle \sin(\phi_{e\ell})\rangle_{\pi^\pm}$

$2\langle \sin(\phi_{\pi\ell})\rangle_{\pi^\pm}$
Contribution by decay of exclusively produced vector mesons is not negligible
Contribution by decay of exclusively produced vector mesons is not negligible.

To evaluate the impact of this contribution on the extracted azimuthal moments, a new observable was regarded which does not experience contributions from the $\rho^0$: the pion-difference target-spin asymmetry

$$A_{UT}^{\pi^+ - \pi^-}(\phi, \phi_S) \equiv \frac{1}{S_T} \frac{(\sigma_{U \uparrow}^{\pi^+} - \sigma_{U \uparrow}^{\pi^-}) - (\sigma_{U \downarrow}^{\pi^+} - \sigma_{U \downarrow}^{\pi^-})}{(\sigma_{U \uparrow}^{\pi^+} - \sigma_{U \uparrow}^{\pi^-}) + (\sigma_{U \downarrow}^{\pi^+} - \sigma_{U \downarrow}^{\pi^-})}$$
Pion-difference asymmetry

\[ A_{UT}^{\pi^+ - \pi^-} (\phi, \phi_S) \equiv \frac{1}{S_T} \left( \frac{\sigma_{U \uparrow}^{\pi^+} - \sigma_{U \uparrow}^{\pi^-}}{\sigma_{U \uparrow}^{\pi^+} - \sigma_{U \uparrow}^{\pi^-}} - \frac{\sigma_{U \downarrow}^{\pi^+} - \sigma_{U \downarrow}^{\pi^-}}{\sigma_{U \downarrow}^{\pi^+} - \sigma_{U \downarrow}^{\pi^-}} \right) \]

Contribution from exclusive $\rho^0$ largely cancels out

Significantly positive amplitudes are obtained as a function of $x, z, P_{h\perp}$.

the underlying (Collins and Sivers) asymmetry amplitudes are not generated by vector meson contribution.
Pion-difference asymmetry

\[
A_{UT}^{\pi^+-\pi^-}(\phi, \phi_S) \equiv \frac{1}{S_T} \frac{(\sigma_{U \uparrow}^{\pi^+} - \sigma_{U \uparrow}^{\pi^-}) - (\sigma_{U \downarrow}^{\pi^+} - \sigma_{U \downarrow}^{\pi^-})}{(\sigma_{U \uparrow}^{\pi^+} - \sigma_{U \uparrow}^{\pi^-}) + (\sigma_{U \downarrow}^{\pi^+} - \sigma_{U \downarrow}^{\pi^-})}
\]

Contribution from exclusive $\rho^0$ largely cancels out

\[
A_{UT}^{\pi^+-\pi^-} = \frac{-4 f_{1T}^{\perp,u} - f_{1T}^{\perp,d}}{4 f_{1T}^{u} - f_{1T}^{d}}
\]

(assuming charge-conjugation and isospin symmetry amongst the pion fragmentation functions)

Significantly positive amplitudes are obtained as a function of $x, z, P_{h \perp}$.

the underlying (Collins and Sivers) asymmetry amplitudes are not generated by vector meson contribution.
\( A_{UT}^{\sin(\phi+\phi_s)} \propto h_1(x) \otimes H_1^{\perp q}(z) \)

\( lp \rightarrow l'hX \)

\( ld \rightarrow l'hX \)
First extraction of transversity distribution

\[ A_{UT}^{\sin(\phi + \phi_s)} \propto h_1(x) \otimes H_{1q}(z) \]

\( lp \to l' hX \)
\( ld \to l' hX \)

Consistent picture

Also with new proton data from COMPASS

(based on Gaussian ansatz)
Extraction of Sivers function

E. Boglione at Transversity 2008
An alternative way to access transversity: the di-hadron SSA

$$ep \rightarrow e'h_1h_2X$$

Interference FF

$$H_{1,q}^{\perp<} (z, M_{\pi\pi}, \cos(\vartheta))$$

(does not depend on quark transv. momentum)

**Chiral-odd T- odd**

Correlation between transverse spin of the fragmenting quark and the relative orbital angular momentum of the hadron pair.

Describes Spin-orbit correlation in fragmentation

azimuthal asymmetries in the direction of the outgoing hadron pairs.
An alternative way to access transversity: the di-hadron SSA

$$\text{ep} \rightarrow e'h_1 h_2 X$$

**Interference FF**

$$H_{1,q}^{\perp\perp} (z, M_{\pi\pi}, \cos(\vartheta))$$

(does not depend on quark transv. momentum)

**Chiral-odd**  **T- odd**

Correlation between transverse spin of the fragmenting quark and the relative orbital angular momentum of the hadron pair.

Describes Spin-orbit correlation in fragmentation

**azimuthal asymmetries** in the direction of the outgoing hadron pairs.

Events are integrated over the transverse momentum of the 2-pion system. Therefore $h_1$ and IFF appear in the cross section in simple direct product.

No convolution integral over transverse momentum involved!
**Di-hadron amplitudes**

- Independent way to access transversity
- No complications due to convolution integral
- ...but limited statistical power (v.r.t. 1-hadron SSAs)

No evidence of the sign-change at the $\rho^0$ mass

Evidence of a T-odd and chiral-odd interference $\mathcal{F} \mathcal{F}!$

\[ \propto h_{1,q}(x) H_{1,q}(z) \neq 0 \]

**JHEP 06 (2008) 017**

Jaffe et al.  
*Phys. Rev. Lett. 80, 1166 (1998)*

Bacchetta and Radici  
*hep-ph/0608037*
Conclusions

• **significant Collins amplitudes observed for** $\pi$-mesons
  → enabled first extraction of transversity

• **significant Sivers amplitudes observed for** $\pi^+$ and $K^+$
  → clear evidence of non-zero Sivers function
  → (indirect) evidence for non-zero quark orbital angular momentum

• Non vanishing pion-difference amplitudes demonstrates that measured asymmetries do not arise from (only) exclusive vector meson contamination

• Current extractions of transversity and Sivers function based on unweighted moments (need Gaussian ansatz)
  • Assumption-free extractions can be achieved in the future from $P_{h \perp}$-weighted moments.

• **significant di-hadron amplitudes observed**
  → clear evidence of non-zero Interference Fragmentation Function
  → more transparent interpretation in terms of DF and FF (no convol. integral)
Back-up slides
2-D moments for $\pi^\pm$: $z$ vs. $P_{h\perp}$

Collins

Sivers
The extraction of the Distribution Functions

\[ \langle \sin(\phi + \phi_S) \rangle^h_{UT} = \frac{\int d\phi_S d^2 \vec{P}_{h\perp} \sin(\phi + \phi_S) \, d\sigma_{UT}}{\int d\phi_S d^2 \vec{P}_{h\perp} d\sigma_{UU}} \approx \frac{\vec{k}_T \cdot \vec{P}_{h\perp}}{M_h} h_1(x, p_T^2) H_1^{q,q}(z, k_T^2) \]

Convolution integral on transverse momenta \( p_T \) and \( k_T \)

\[ \langle \sin(\phi - \phi_S) \rangle^h_{UT} = \frac{\int d\phi_S d^2 \vec{P}_{h\perp} \sin(\phi - \phi_S) \, d\sigma_{UT}}{\int d\phi_S d^2 \vec{P}_{h\perp} d\sigma_{UU}} \approx \frac{\vec{p}_T \cdot \vec{P}_{h\perp}}{M} f_{1T}^{q,q}(x, p_T^2) D_1^q(z, k_T^2) \]

**Experiment:** only partial coverage of the full \( P_{h\perp} \) range (acceptance effects)

**Theory:** difficult to solve \( \iff \) Gaussian ansatz

\[ h_1(x, p_T^2) \approx \frac{h_1(x)}{\pi \langle p_T^2(x) \rangle} e^{-\frac{p_T^2}{\langle p_T^2(x) \rangle}} \quad \text{and} \quad H_1^{q,q}(z, k_T^2) \approx \frac{H_1^{q,q}(z)}{\pi \langle k_T^2(z) \rangle} e^{-\frac{k_T^2}{\langle k_T^2(z) \rangle}} \]

(extraction assumption-dependent)
Alternatively one can use the so-called $P_{h\perp}$-weighted moments (don’t require any assumption on transverse momenta distributions)

$$
\left< \frac{P_{h\perp}}{zM} \sin(\phi - \phi_S) \right>_\text{UT}^h \equiv \int d\phi_S d^2 \vec{P}_{h\perp} \sin(\phi - \phi_S) \frac{P_{h\perp}}{zM} d^6 \sigma_{\text{UT}} \int d\phi_S d^2 \vec{P}_{h\perp} d^6 \sigma_{\text{UU}}
$$

$P_{hT}$-weighted Sivers moments (measured)

$$
P^h_q(x, z) \equiv \frac{e^2_q q(x) D_{1}^{q\rightarrow h}(z)}{\sum_{q' \bar{q}'} e^2_{q'} q'(x) D_{1}^{q'\rightarrow h}(z)} \quad \text{purities (based on known quantities)}
$$

Extraction above requires, in principle, a full integration over $P_{h\perp}$ (from 0 to $\infty$)

Due to the partial experimental coverage in $P_{h\perp}$ the evaluation of acceptance effects is of crucial importance.