Direct Extraction of Helicity Amplitude Ratios in Exclusive $\rho^0$ Electroproduction

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Physics Motivation

\[ \gamma^* + N \rightarrow V + N \] is a perfect reaction to study both vector-meson production mechanism and hadron structure. Spin Density Matrix Elements (SDMEs) of \( \rho^0 \) at HERMES: EPJ C62 (2009) 659. SDMEs are expressible in terms of ratios of helicity amplitudes, hence ratios can be extracted from angular distribution of decay \( \pi^+ \pi^- \).

- Data on \( \frac{d\sigma}{dt} = \sum |F_{\lambda_V \lambda'_N \lambda \gamma \lambda_N}|^2 \) additional to SDMEs gives a possibility to extract moduli of all the helicity amplitudes and phase differences between them.

- Generalized Parton Distributions (GPDs) of the nucleon can be obtained from the amplitude \( F_{00} \equiv F_{0^{1/2}0^{1/2}} (\gamma_L \rightarrow V_L) \) for which factorization theorem is proved. Extraction of amplitude ratios is a first step to get \( F_{00} \) and GPDs.

- Difference between proton and deuteron results would points out contribution of \( q\bar{q} \)-exchange with isospin \( I = 1 \) and natural parity \( P = (-1)^J \) (\( \rho, a_0, a_2 \) reggeons).
Extraction of amplitude ratios provides a possibility to distinguish between contributions of Natural Parity Exchange (NPE, $J^P = 0^+, 1^-$, ...) amplitudes $T_{\lambda_V \lambda_\gamma}$ (Pomeron = two-gluon exchange, $\rho$, $\omega$, $a_2$,... reggeons = $q\bar{q}$ exchange) and Unnatural Parity Exchange (UPE, $J^P = 0^-, 1^+$, ...) amplitudes ($\pi$, $a_1$, $b_1$,... reggeons = $q\bar{q}$ exchange) $U_{\lambda_V \lambda_\gamma}$ better than in SDME method.

Violation of $s$-channel helicity ($\lambda_V \neq \lambda_\gamma$) can be studied more reliably on the language of amplitude ratios rather than in SDME analysis. Spin-flip amplitudes $T_{01}, T_{10}$ provide information on vector-meson structure. They are zero in the absence of quark motion in vector mesons (if quark carries momentum fraction $z = \frac{1}{2}$).
Kinematics of Exclusive $\rho^0$-Meson Production at HERMES

- $W = 3.0 \div 6.5$ GeV, $< W >= 4.9$ GeV Total number of events (1996-2005)
- $Q^2 = 0.5 \div 7.0$ GeV$^2$, $< Q^2 >= 1.95$ GeV$^2$ Deuteron: $\rho^0$ - 16388
- $x_B = 0.01 \div 0.35$, $< x_B >= 0.08$ Hydrogen: $\rho^0$ - 9860
- $0 \leq -t' \leq 0.4$ GeV$^2$, $< -t' >= 0.13$ GeV$^2$ with $t' = t - t_{min}$

$$\Delta E = \frac{M_X^2 - M_p^2}{2M_p}$$ with $M_X^2 = (p + q - p_{\pi^+} - p_{\pi^-})^2$ and $M_X$ being missing mass

$-1.0 < \Delta E < 0.6$ GeV, SIDIS background is subtracted with the help of MC (PYTHIA)

$0.6 < M_{\pi\pi} < 1$ GeV,
Data Processing using Maximum Likelihood Method in MINUIT

\[ \Psi = \phi - \Phi \text{ (in } S\text{-Channel Helicity Conservation (SCHC) approximation)} \]

- **Monte Carlo Events:** 3-dimensional matrix of fully reconstructed MC events at initial uniform angular distribution.
- **Binned Maximum Likelihood (BML) Method:** $8 \times 8 \times 8$ bins of $\cos(\Theta), \phi, \Phi$. Simultaneous fit of 23 SDMEs (5 ratios of helicity amplitudes) for data with negative and positive beam helicity ($<P_b> = \pm 53.5\%$) and unpolarized target. Agreement of fitted angular distributions with the HERMES data.
Amplitude Method and Spin-Density Matrix Element Method

- **First:** $e \rightarrow e + \gamma^*$ (QED)
  Spin-Density Matrix (SDM) of the virtual photon $\rho(\Phi, \epsilon)$

- **Second:** $\gamma^* + N \rightarrow V + N$ (QCD)
  Helicity amplitudes in CM system of $\gamma^* N F_{\lambda V} \lambda_N'; \lambda \gamma \lambda_N (W, Q^2, t')$
  Vector-meson spin-density matrix
  \[ r = \frac{1}{2N} \text{tr} \lambda_N \lambda_N' \{ F \rho F^+ \}, \]
  \[ N = \text{Tr} \lambda_N \lambda_N' \{ F \rho F^+ \}. \]
  If SDM of $\gamma^*$ is decomposed into set of nine matrices $\Sigma^\alpha$ then SDMEs are
  \[ r_{\lambda V}^{\alpha} = \frac{1}{2N} \text{tr} \lambda_N \lambda_N' \{ F \Sigma^\alpha F^+ \} \lambda_V \lambda_V'. \]

- **Third:** $\rho^0 \Rightarrow \pi^+ \pi^-$ (conservation of $\vec{J}$)
  $|\rho^0; 1m \rangle \rightarrow |\pi^+ \pi^-; 1m \rangle \Rightarrow Y_{1m}(\theta, \phi)$
  Angular distribution $\mathcal{W}(\Phi, \phi, \cos \Theta)$ depends linearly on $r_{\lambda V}^{\alpha}$ and $P_b$.

SDME method

- 23 LU SDMEs (for Longitudinally polarized beam and Unpolarized target) are considered as free parameters in fit of angular distribution of pions from decay $\rho^0 \rightarrow \pi^+ + \pi^-$ in any small bin of kinematic variables ($Q^2, t'$ etc.).

Relation of SDMEs and helicity amplitudes is ignored.

Amplitude method

- SDMEs are expressed in terms of ratios of helicity amplitudes.
- Helicity amplitude ratios are free parameters in fit of angular distribution in any small bin.
- Binning $4 \times 4$ of $Q^2$ and $-t'$. 
Amplitude Method

- 18 independent amplitudes
  34 real free parameters (functions)
- 23 LU SDMEs (< 34)
- Hierarchy of amplitudes at small $t'$ and high $Q^2$.
  Neglect small amplitudes.
- NPE ($T_{\lambda V\lambda' N\lambda' N\lambda N}$) and UPE ($U_{\lambda V\lambda' N\lambda' N\lambda N}$) helicity amplitudes.
  $F = T + U$,
  $T/U_{\lambda V\lambda' N\lambda' N\lambda N} = \frac{1}{2}(F_{\lambda V\lambda' N\lambda' N\lambda N}$
  $\pm (-1)^{\lambda N - \lambda'_N}F_{\lambda V' - \lambda'_N\lambda N}$)
  Shorthand notation:
  $T_{\lambda V\lambda N} = T_{\lambda V\frac{1}{2}\lambda\frac{1}{2}}$
- No interference between NPE and UPE amplitudes for LU SDMEs
- UPE are suppressed at high $W$.
  Neglect all UPE amplitudes?
- No interference between amplitudes with and without nucleon spin flip.
  $T_{\lambda' N\lambda N}/T_{\lambda' N=\lambda N} \sim \alpha = v_T/(2M)$.
  Fractional contribution of NPE amplitudes with $\lambda'_N \neq \lambda_N$ to LU SDMEs
  $\sim \alpha^2 < \text{experimental uncertainty}$.
- Neglect with NPE nucleon spin-flip amplitudes retains $T_{11}/T_{00}, T_{01}/T_{00}, T_{10}/T_{00}, T_{1-1}/T_{00}$ (8 parameters).
- SDME analysis: $S$-channel helicity conservation (SCHC) at small $t'$.
  $|U_{01}|, |U_{10}|, |U_{1-1}| \ll |U_{11}|$ retains only
  $|U_{11}| = \sqrt{|U_{1\frac{1}{2}\frac{1}{2}}|^2 + |U_{1-\frac{1}{2}\frac{1}{2}}|^2}$
  9th parameter: $|U_{11}|/|T_{00}|$.
- Hierarchy of extracted amplitudes at HERMES kinematic region
  $|T_{00}|^2 \sim |T_{11}|^2 \gg |U_{11}|^2 > |T_{01}|^2 > |T_{10}|^2 \sim |T_{1-1}|^2$
- No difference between proton and deuteron results for amplitude ratio $T_{11}/T_{00}$.
- pQCD prediction (Ivanov, Kirshner; Kuraev, Nikolaev, Zakharov): $T_{11}/T_{00} \propto M_{\rho}/Q$.
- Fit of $Q$ dependence: $\text{Re}(T_{11}/T_{00}) = a/Q$, $\text{Im}(T_{11}/T_{00}) = b \cdot Q$.
  Combined data on proton and deuteron: $a = 1.129 \pm 0.024$ GeV, $\chi^2/N_{df} = 1.02$; $b = 0.344 \pm 0.014$ GeV$^{-1}$, $\chi^2/N_{df} = 0.87$.
- Behaviour of $\text{Im}(T_{11}/T_{00})$ is in a contradiction with high-$Q$ asymptotic in pQCD. Phase difference $\delta_{11} \sim 30^\circ$ and grows with $Q^2$ in disagreement with pQCD calculation.
- No $t$ dependence: difference of slopes $\beta_L - \beta_T = -0.6 \pm 0.4$ GeV$^{-2}$.
• Violation of $S$-Channel Helicity ($\lambda_V \neq \lambda_\gamma$): $T_{01} \neq 0$.
• No difference between proton and deuteron results for amplitude ratio $T_{01}/T_{00}$.
• pQCD prediction (Ivanov, Kirshner; Kuraev, Nikolaev, Zakharov): $\frac{T_{01}}{T_{00}} \propto \frac{\sqrt{-t'}}{Q}$.
• Fit of $t'$ dependence: $\text{Re}(T_{01}/T_{00}) = a\sqrt{-t'}$, $\text{Im}(T_{01}/T_{00}) = b\sqrt{-t'}/Q$.

Combined proton and deuteron data: $a = 0.399 \pm 0.023$ GeV$^{-1}$, $\chi^2/N_{df} = 0.72$; $b = 0.20 \pm 0.07$, $\chi^2/N_{df} = 1.09$. 
No difference between proton and deuteron results for amplitude ratio $|U_{11}/T_{00}|$.

pQCD prediction: $U_{11}/T_{00} \propto M_\rho/Q$.

Neither $Q^2$ nor $t'$ dependence: $|U_{11}|/|T_{00}| = a$, $a = 0.391 \pm 0.013$, $\chi^2/N_{df} = 0.44$

where $|U_{11}|^2 = |U_{1\frac{1}{2}1\frac{1}{2}}|^2 + |U_{1\frac{1}{2}1\frac{1}{2}}|^2$.

Unnatural Parity Exchange is seen much better than in SDME method.

Contradiction both with high-Q asymptotic and one-pion-exchange dominance.
Test of Unnatural-Parity Exchange for $\rho^0$ Meson

- Natural and Unnatural Parity Exchanges in the $t$-channel
  NPE: GPD $H, E$ ; $T_{\lambda \rho \lambda \gamma}$
  UPE: GPD $\tilde{H}, \tilde{E}$ ; $U_{\lambda \rho \lambda \gamma}$
  NPE (Pomeron, $\rho$, $\omega$, $f_2$, $a_2$, ...) dominate and UPE ($\pi$, $a_1$, $b_1$...) are suppressed at high energies
- Signal of UPE in SDME method

$u_1 = 1 - r_{00}^{04} + 2r_{11}^{04} - 2r_{11}^{1} - 2r_{11}^{-1}$,

$u_1 = \sum \lambda_N \lambda'_N \frac{2\epsilon|U_{10}|^2 + |U_{11} + U_{-11}|^2}{N}$

where $N = N_T + \epsilon N_L$,
$N_T = \sum \lambda_N \lambda'_N (|T_{11}|^2 + |T_{01}|^2 + |T_{-11}|^2 + |U_{11}|^2 + |U_{01}|^2 + |U_{-11}|^2)$,
$N_L = \sum \lambda_N \lambda'_N (|T_{00}|^2 + |T_{10}|^2 + |T_{-10}|^2 + |U_{10}|^2 + |U_{-10}|^2)$.

$u_1 = 0.125 \pm 0.021_{\text{stat}} \pm 0.050_{\text{syst}}$ (H),
$u_1 = 0.091 \pm 0.016_{\text{stat}} \pm 0.046_{\text{syst}}$ (D)
$u_1 = 0.106 \pm 0.036_{\text{tot}}$ (H+D)

World Results on Ratios of Helicity Amplitudes

- **H1**: Unpolarized beam and unpolarized target (15 SDMEs), \( < Q^2 > = 3.3 \text{ GeV}^2 \).
- Additional assumption: all amplitudes are imaginary, all amplitude ratios are real.
- **HERMES**: Longitudinally polarized beam and unpolarized target (23 SDMEs). Both real and imaginary parts of ratios of helicity amplitudes are extracted.
- Excellent agreement of amplitude ratios extracted by H1 and HERMES.
Summary

- Measurement of $\rho^0$-meson production by longitudinally polarized electron/positron beam on unpolarized proton and deuteron in the HERMES experiment permits to extract both real and imaginary parts of $T_{11}/T_{00}$, $T_{01}/T_{00}$, $T_{10}/T_{00}$, $T_{1-1}/T_{00}$, and $|U_{11}/T_{00}|$.
- Dependences of the most reliably obtained ratios $T_{11}/T_{00}$, $T_{01}/T_{00}$, $|U_{11}/T_{00}|$ on $Q^2$ and $t'$ is studied. The observed dependences of $\text{Im}(T_{11}/T_{00})$ and $|U_{11}/T_{00}|$ are in contradiction with high-$Q$ asymptotic behaviour predicted in pQCD while dependences of $\text{Re}(T_{11}/T_{00})$ and $\text{Im}(T_{01}/T_{00})$ are in agreement with pQCD prediction.
- No statistically significant difference between proton and deuteron results for amplitude ratios $T_{11}/T_{00}$, $T_{01}/T_{00}$, $|U_{11}/T_{00}|$ is found.
- Violation of $S$-channel helicity is observed in amplitude method with higher accuracy than in SDME method.
- Contribution of unnatural parity exchange amplitude $U_{11}$ of $\rho^0$-meson production is found in amplitude method with much higher accuracy than in SDME analysis.

Outlook

- To decrease background contribution by measuring recoil nucleon.
- Include data on transversely polarized target into amplitude analysis.