Spin Density Matrix Elements from diffractive $\phi$ vector meson production at HERMES

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Rudiments

Spin Density Matrix Elements (SDME’s) : definitions and their determination

The derived observables:

- SDME’s and Amplitudes vector mesons
- Dependences of SDME’s on $Q^2$ and $t'$
- $R = \frac{\sigma_L}{\sigma_T}$
- the signatures of the Natural or Unnatural Parity Exchange amplitudes
- The Transverse Target Spin Asymmetry - $A_{UT}$

Conclusions
\[ e + p \rightarrow e' + p' + V: \text{Rudiments} \]

- In one photon approximation
  \[ \equiv \gamma^* + p \rightarrow p' + V \]
- The amplitude of this process can be factorized:
  \[ A = \Phi_{\gamma^* \rightarrow q\bar{q}} \otimes A_{q\bar{q} + p \rightarrow q\bar{q} + p} \otimes \Phi_{q\bar{q} \rightarrow V}. \]
- In these three steps the interaction time \((q\bar{q})\) with target is shorter than \(\gamma^*\) fluctuation and formation of VM. (Collins, Frankfurt and Strikman Phys. Rev D56(1997)2982)
- \(\gamma^* + N \rightarrow \phi^0 + N'\) is good tool to study the helicity conservation:
  - helicity state of \(\gamma^*\) is easy to determine (QED)
  - \(\phi^0 \rightarrow K^+K^-\) decay determines the helicity of \(\phi^0\)

**Kinematics:**
- \(\nu = 5 \div 24 \text{ GeV}, <\nu> = 13.3 \text{ GeV},\)
- \(Q^2 = 0.5 \div 7.0 \text{ GeV}^2, <Q^2> = 2.3 \text{ GeV}^2\)
- \(W = 3.0 \div 6.5 \text{ GeV}, <W> = 4.9 \text{ GeV},\)
- \(x_{Bj} = 0.01 \div 0.35, <x_{Bj}> = 0.07\)
- \(t' = (t - t_{min})\)
  - \(t' = 0 \div 0.4 \text{ GeV}^2, <t'> = 0.13 \text{ GeV}^2\)
$e + p \rightarrow e' + p' + \phi \rightarrow K^+ K^-$

The exclusive events were selected from the missing energy spectra \( \Delta E = \frac{M_X^2 - M_p^2}{2M_p} \).

The background was simulated by code MC PYTHIA.
**Φ-meson Spin Density Matrix Elements (SDMEs)**

SDMEs: $r_{\lambda V, \lambda' V}^{\alpha} \sim \rho(V) = \frac{1}{N} \sum_{\lambda', \lambda, \gamma} (T_{\lambda V, \lambda, \gamma} \rho(\gamma) T_{\lambda' V, \lambda', \gamma}^{+})$

spin-density matrix of the vector meson $\rho(V)$ in terms of the photon matrix $\rho(\gamma)$ and helicity amplitude $T_{\lambda V, \lambda, \gamma}$

presented according to K.Schilling and G.Wolf (Nucl. Phys. B61 (1973) 381)

$\alpha = 04$ - long. or trans. photon with $\lambda_\phi = 0$; $\alpha = 1-2$ - trans. with lin. pol.;

$\alpha = 3$ - trans. with cir. pol.; $\alpha = 5-8$ - interf. trans./long. terms.

measured experimentally at $5 < W < 75$ GeV (HERMES,COMPASS,H1,ZEUS)

provide access to helicity amplitudes $T_{\lambda V, \lambda, \gamma}$ and phases, which are:

- extracted experimentally from SDMEs
Angular Distributions Fit: Likelihood Method in MINUIT

Simulated Events: matrix of fully reconstructed MC events from initial uniform angular distribution

Binned Maximum Likelihood Method: $8 \times 8 \times 8$ bins of $\cos(\Theta), \phi, \Phi$. Simultaneous fit of 23 SDMEs $r_{ij}^\alpha = W(\Phi, \phi, \cos \Theta)$ for data with negative and positive beam helicity ($< |P_b| > = 53.5\%, \Psi = \Phi - \phi$)

$\Leftarrow$ red line after the fit MAX. Likelihood method, black one starting parameters
Function for the Fit of 23 SDME $r_{ij}^\alpha$

$$W(\cos \Theta, \phi, \Phi) = W^{unpol} + W^{long.pol},$$

$$W^{unpol}(\cos \Theta, \phi, \Phi) = \frac{3}{8\pi^2} \left[ \frac{1}{2} (1 - r_{00}^{04}) + \frac{1}{2} (3r_{00}^{04} - 1) \cos^2 \Theta - \sqrt{2} \text{Re}\{r_{10}^{04}\} \sin 2\Theta \cos \phi - r_{1-1}^{04} \sin^2 \Theta \cos 2\phi 
- \epsilon \cos 2\Phi \left( r_{11}^{1} \sin^2 \Theta + r_{00}^{1} \cos^2 \Theta - \sqrt{2} \text{Re}\{r_{10}^{1}\} \sin 2\Theta \cos \phi - r_{1-1}^{1} \sin^2 \Theta \cos 2\phi \right) 
- \epsilon \sin 2\Phi \left( \sqrt{2} \text{Im}\{r_{10}^{2}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{2}\} \sin^2 \Theta \sin 2\phi \right) 
+ \sqrt{2}\epsilon(1 + \epsilon) \cos \Phi \left( r_{11}^{5} \sin^2 \Theta + r_{00}^{5} \cos^2 \Theta - \sqrt{2} \text{Re}\{r_{10}^{5}\} \sin 2\Theta \cos \phi - r_{1-1}^{5} \sin^2 \Theta \cos 2\phi \right) 
+ \sqrt{2}\epsilon(1 + \epsilon) \sin \Phi \left( \sqrt{2} \text{Im}\{r_{10}^{6}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{6}\} \sin^2 \Theta \sin 2\phi \right) \right],$$

$$W^{long.pol.}(\cos \Theta, \phi, \Phi) = \frac{3}{8\pi^2} P_{beam} \left[ \sqrt{1 - \epsilon^2} \left( \sqrt{2} \text{Im}\{r_{10}^{3}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{3}\} \sin^2 \Theta \sin 2\phi \right) 
+ \sqrt{2}\epsilon(1 - \epsilon) \cos \Phi \left( \sqrt{2} \text{Im}\{r_{10}^{7}\} \sin 2\Theta \sin \phi + \text{Im}\{r_{1-1}^{7}\} \sin^2 \Theta \sin 2\phi \right) 
+ \sqrt{2}\epsilon(1 - \epsilon) \sin \Phi \left( r_{11}^{8} \sin^2 \Theta + r_{00}^{8} \cos^2 \Theta - \sqrt{2} \text{Re}\{r_{10}^{8}\} \sin 2\Theta \cos \phi - r_{1-1}^{8} \sin^2 \Theta \cos 2\phi \right) \right].$$
The SDME’s for proton (red) and deuteron (blue) for $1.0 < Q^2 < 7.0 \text{ GeV}^2$. 
SDMEs and Amplitudes

A- SCHC \( \gamma_L^* \rightarrow \phi_L^0 \) and \( \gamma_T^* \rightarrow \phi_T^0 \)

\[ |T_{11}|^2 \propto 1 - r_{00}^{04} \propto r_{1-1}^{1} \propto -\text{Im}\{r_{1-1}^{2}\} \]

B- Interference: \( \gamma_L^*, \phi_T^0 \)

\[
\begin{align*}
\text{Re}\{T_{00}T_{11}^*\} & \propto \text{Re}\{r_{10}^{5}\} \propto -\text{Im}\{r_{10}^{6}\} \\
\text{Im}\{T_{11}T_{00}^*\} & \propto \text{Im}\{r_{10}^{7}\} \propto \text{Re}\{r_{10}^{8}\}
\end{align*}
\]

C- Spin Flip: \( \gamma_T^* \rightarrow \phi_L^0 \)

\[
\begin{align*}
\text{Re}\{T_{11}T_{01}^*\} & \propto \text{Re}\{r_{10}^{04}\} \\
& \propto \text{Re}\{r_{10}^{1}\} \propto \text{Im}\{r_{10}^{2}\} \\
\text{Re}\{T_{01}T_{00}^*\} & \propto r_{00}^{5} \\
|T_{01}|^2 & \propto r_{00}^{1} \\
\text{Im}\{T_{01}T_{11}^*\} & \propto \text{Im}\{r_{10}^{3}\} \\
\text{Im}\{T_{01}T_{00}^*\} & \propto r_{00}^{8}
\end{align*}
\]

D-Spin Flip: \( \gamma_L^* \rightarrow \phi_T^0 \)

\[
\begin{align*}
\text{Re}\{T_{10}T_{11}^*\} & \propto r_{11}^{5} \propto r_{1-1}^{5} \propto \text{Im}\{r_{1-1}^{6}\} \\
\text{Im}\{T_{10}T_{11}^*\} & \propto \text{Im}\{r_{1-1}^{7}\} \propto r_{11}^{8} \propto r_{1-1}^{8}
\end{align*}
\]

E- Double Spin Flip: \( \gamma_T^* \rightarrow \phi_{-T}^0 \)

\[
\begin{align*}
\text{Re}\{T_{-1-1}T_{11}^*\} & \propto r_{-1-1}^{04} \propto r_{11}^{1} \\
\text{Im}\{T_{-1-1}T_{11}^*\} & \propto \text{Im}\{r_{1-1}^{3}\}
\end{align*}
\]

The phase difference \( \delta_{p+d}^{\phi} \) between transverse \( T_{11} \) and \( T_{00} \) amplitudes was determined:

\[
tg(\delta^\phi) = \frac{\text{Im}r_{10}^{7} - \text{Re}r_{10}^{8}}{\text{Re}r_{10}^{5} - \text{Im}r_{10}^{6}}, \delta_{p+d}^{\phi} = 33.0^0 \pm 7.4^0.
\]
A- SCHC $\gamma_L^* \rightarrow \phi_L^0$ and $\gamma_T^* \rightarrow \phi_T^0$

$|T_{11}|^2 \propto 1 - r_{00}^{04} \propto r_{1-1}^1 \propto -\text{Im}\{r_{1-1}^2\}$

B- Interference: $\gamma_L^*, \phi_T^0$

$\text{Re}\{T_{00}T_{11}^*\} \propto \text{Re}\{r_{10}^5\} \propto -\text{Im}\{r_{10}^6\}$

$\text{Im}\{T_{11}T_{00}^*\} \propto \text{Im}\{r_{10}^7\} \propto \text{Re}\{r_{10}^8\}$

C- Spin Flip: $\gamma_T^* \rightarrow \phi_L^0$

$\text{Re}\{T_{11}T_{01}^*\} \propto \text{Re}\{r_{10}^{04}\}$

$\propto \text{Re}\{r_{10}^1\} \propto \text{Im}\{r_{10}^2\}$

$\text{Re}\{T_{01}T_{10}^*\} \propto r_{00}^5$

$|T_{01}|^2 \propto r_{00}^1$

$\text{Im}\{T_{01}T_{11}^*\} \propto \text{Im}\{r_{10}^3\}$

$\text{Im}\{T_{01}T_{00}^*\} \propto r_{00}^8$

D- Spin Flip: $\gamma_L^* \rightarrow \phi_T^0$

$\text{Re}\{T_{10}T_{11}^*\} \propto r_{11}^5 \propto r_{1-1}^5 \propto \text{Im}\{r_{1-1}^6\}$

$\text{Im}\{T_{10}T_{11}^*\} \propto \text{Im}\{r_{1-1}^7\} \propto r_{11}^8 \propto r_{1-1}^8$

E- Double Spin Flip: $\gamma_T^* \rightarrow \phi_{-T}^0$

$\text{Re}\{T_{1-1}T_{11}^*\} \propto r_{1-1}^{04} \propto r_{11}^1$

$\text{Im}\{T_{1-1}T_{11}^*\} \propto \text{Im}\{r_{1-1}^3\}$

$\Rightarrow$ Hierarchy of $\phi^0$ amplitudes:

$|T_{00}| \sim |T_{11}| \gg |T_{01}| > |T_{10}| \gtrsim |T_{1-1}|, (0 \rightarrow L, 1 \rightarrow T)$

$\Rightarrow \phi$ meson SDMEs are consistent with SCHC, $|T_{00}| \sim |T_{11}|$
Dependences of $\phi$ meson SDME's on $Q^2$

The dependences of SDME's on $Q^2$ for proton and deuteron data. The outer bars represent the total, the inner ones the statistical errors.

**INDICATIONS:**

- **green:** Helicity conserving transitions (SCHC) - $\gamma^*_L \rightarrow V_L$, $\gamma^*_T \rightarrow V_T$
- **yellow:** Single Flip - $\gamma^*_T \rightarrow V_L$
- **blue:** Single Flip - $\gamma^*_L \rightarrow V_T$
- **blank:** Double Flip - $\gamma^*_T \rightarrow V_{-T}$
Dependences of $\phi$ meson SDME's on $t'$

The dependences of SDME's on $t'$ for proton and deuteron data. The outer bars represent the total, the inner ones the statistical errors.

**INDICATIONS:**

- **green:** Helicity conserving transitions (SCHC) - $\gamma_{L}^{*} \rightarrow V_{L}, \gamma_{T}^{*} \rightarrow V_{T}$
- **yellow:** Single Flip - $\gamma_{T}^{*} \rightarrow V_{L}$
- **blue:** Single Flip - $\gamma_{L}^{*} \rightarrow V_{T}$
- **blank:** Double Flip - $\gamma_{T}^{*} \rightarrow V_{-T}$
Comparison of commonly measured:

\[ R^{04} = \frac{\sigma_L}{\sigma_T} = \frac{1}{\epsilon} \frac{r^{04}_{00}}{1 - r^{04}_{00}}, \]

where:

\[ r^{04}_{00} = \sum \{ \epsilon |T_{00}|^2 + |T_{11}|^2 \}/\sigma_{tot} \]

\[ \sigma_{tot} = \epsilon \sigma_L + \sigma_T \]

\[ \implies R^{04} \text{ for } \phi \text{ meson at HERMES is in good agreement with world data.} \]
Natural Parity Exchange in $\phi$ Meson Leptoproduction

\[ U_1 = 0.02 \pm 0.07_{\text{stat}} \pm 0.16_{\text{syst}} \]
\[ U_2 = -0.03 \pm 0.01_{\text{stat}} \pm 0.03_{\text{syst}} \]
\[ U_3 = -0.05 \pm 0.11_{\text{stat}} \pm 0.07_{\text{syst}} \]

⇒ no UPE for $\phi$ meson production, as expected
Transverse Target Spin Asymmetry (TTSA)

TTSA allows to separate, sensitive to helicity-flip E GPDs with different spin dependence which contain information about the orbital angular momentum.

Def: \( A_{UT} = \frac{d\sigma(\phi_s) - d\sigma(\phi_s + \pi)}{d\sigma(\phi_s) + d\sigma(\phi_s + \pi)} \), for \( P_T = 1, \ P_L = 0 \)

Def: \( A_{UT}^{*} = \frac{d\sigma(\phi_s) - d\sigma(\phi_s + \pi)}{d\sigma(\phi_s) + d\sigma(\phi_s + \pi)} \), for \( S_T = 1, \ S_L = 0 \)

Determined following prescription of M. Diehl and S. Sapeta: hep-ph/0503023v1

from angular distribution \( W(P_T, \cos(\theta), \phi, \phi_s) \) as amplitudes of \( \sin(\phi \pm \phi_s) \).
Conclusions

The first measurement of the complete set of SDMEs for $\phi$ mesons

- The transitions $\gamma^*_L \rightarrow \phi^*_L$ and $\gamma^*_T \rightarrow \phi^*_T$ are dominant.
  SDME's: $(1 - r_{00}^0), \ r_{1-1}^1, \ \text{Im} \ r_{1-1}^2$, depend on $Q^2$, i.e. $\sim \frac{1}{Q^2 + m_v^2}$

- The determined value of the difference between phase transitions $\gamma^*_L \rightarrow \phi^*_T$ and $\gamma^*_T \rightarrow \phi^*_L$
  $\delta_{p+d}^\phi = 33.00 \pm 7.40$.

- The SDME’s describing the single-helicity-flip transitions: $\gamma^*_T \rightarrow \phi^*_L$ and $\gamma^*_L \rightarrow \phi^*_T$ as well as the double-helicity-flip $\gamma^*_T \rightarrow \phi^*_T$ fluctuate near zero values.

- Dependence on target (H, D): not observed.

- Only Natural-Parity Exchange was observed.

- The comparisons of the $\frac{\sigma_L}{\sigma_T}$ with other measurements: good agreement.

- Determination of TTSA with L,T separation

The latest theoretical calculations in GPD model: S.V. Goloskokov, P. Kroll arXiv:0708.3569 [hep-ph] 27.08.07, have been done for HERMES kinematics.
Dependencies of $\rho^0$ meson SDME's on $Q^2$

INDICATIONS:
- green: SCHHC - $\gamma^*_L \rightarrow V_L, \gamma^*_T \rightarrow V_T$
- yellow: Single Flip - $\gamma^*_T \rightarrow V_L$
- grid: Single Flip - $\gamma^*_L \rightarrow V_T$
- blank: Double Flip - $\gamma^*_T \rightarrow V_{-T}$

HERMES PRELIMINARY
- ■ proton
- ● deuteron
Dependencies of $\rho^0$ meson SDME’s on $t'$

INDICATIONS:
- green: SCHHC - $\gamma^*_L \rightarrow V_L, \gamma^*_T \rightarrow V_T$
- yellow: Single Flip - $\gamma^*_T \rightarrow V_L$
- grid: Single Flip - $\gamma^*_L \rightarrow V_T$
- blank: Double Flip - $\gamma^*_T \rightarrow V_{-T}$
Comparison of commonly measured:

\[ R^{04} = \frac{1}{\epsilon} \frac{r^{04}}{1 - r^{04}} , \]

\[ r^{04} = \sum \{ \epsilon \left| T_{00} \right|^2 + \left| T_{01} \right|^2 + \left| U_{01} \right|^2 \} / \sigma_{tot} , \]

\[ \sigma_{tot} = \epsilon \sigma_L + \sigma_T , \]

\[ \sigma_T = \sum \{ \left| T_{11} \right|^2 + \left| T_{01} \right|^2 + \left| T_{1-1} \right|^2 + \left| U_{11} \right|^2 \} , \]

\[ \sigma_L = \sum \{ \left| T_{00} \right|^2 + 2 \left| T_{10} \right|^2 \} . \]

Due to the helicity-flip and unnatural parity amplitudes, \( R^{04} \) depends on kinematic conditions, and is not identical to \( R \equiv \left| T_{00} \right|^2 / \left| T_{11} \right|^2 \) at SCHC and NPE dominance.

\[ \Rightarrow \text{Second order contribution of spin-flip amplitudes to } R^{04} \]

\[ \Rightarrow \text{HERMES } \rho^0 \text{ data on } R^{04} \text{ are suggestive to } R(W)-dependence \]
Unnatural Parity Exchange (UPE) in $\rho^0$ Leptoproduction

- Natural-parity exchange: interaction is mediated by a particle of ‘natural’ parity: vector or scalar meson:
  \[ J^P = 0^+, 1^- \text{ e.g. } \rho^0, \omega, a_2 \]

- Unnatural parity exchange is mediated by pseudoscalar or axial meson:
  \[ J^P = 0^-, 1^+ \text{ e.g. } \pi, a_1, b_1 \rightarrow \text{ only quark-exchange contribution} \]

- No interference between NPE and UPE contributions on unpolarized target

- Extracted from SDMEs:
  \[ U_2 + iU_3 \propto (U_{11} + U_{1-1}) \star U_{10} \]
  \[ U_2 = r_{11}^5 + r_{1-1}^5 \]
  \[ \text{p: } U_2 = -0.012 \pm 0.006_{\text{stat}} \pm 0.012_{\text{syst}} \]
  \[ \text{d: } U_2 = -0.008 \pm 0.0046_{\text{stat}} \pm 0.010_{\text{syst}} \]
  \[ U_3 = r_{11}^5 + r_{1-1}^5 \]
  \[ \text{p: } U_3 = -0.020 \pm 0.050_{\text{stat}} \pm 0.007_{\text{syst}} \]
  \[ \text{d: } U_3 = -0.021 \pm 0.038_{\text{stat}} \pm 0.011_{\text{syst}} \]

\[ U_1 \propto \epsilon |U_{10}|^2 + 2 |U_{11} + U_{1-1}|^2 \]
\[ U_1 = 1 - r_{00}^{04} + 2r_{1-1}^{04} - 2r_{11}^{1} - 2r_{-1-1}^{1} \]
\[ \text{p: } U_1 = 0.132 \pm 0.026_{\text{st}} \pm 0.053_{\text{syst}} \]
\[ \text{d: } U_1 = 0.094 \pm 0.020_{\text{st}} \pm 0.044_{\text{syst}} \]
\[ \text{p+d: } U_1 = 0.109 \pm 0.037_{\text{tot}} \]

\[ \implies \text{Indication on hierarchy of } \rho^0 \text{ UPE amplitudes: } |U_{11}| \gg |U_{10}| \sim |U_{01}| \]